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THE FRANKLIN INSTITUTE.

Stated Meeting, held Wednesday, June 17, 1903.

Controlling the Floods of the Mississippi River.

BY PROF. LEWIS M. HAUPT.

Mr. President, Ladies and Gentlemen:

The subject assigned by your committee for this evening is a large one, so that in the time at command it will only be possible to treat of a few generalities. Your attention will therefore be directed chiefly to the humane aspects of the problem, followed by some of the physical conditions which characterize the progress of the improvements, with their results and suggestions as to the remedies which may be applied to still further alleviate the disasters which are all too frequent. Already this season there have been two severe inundations which have risen higher than any previous records, causing great destruction of life and property; but such losses are not unprecedented, for in 1897 the *Public Ledger* said, editorially:

"One of the greatest problems that confronts the Government is the control of the lower Mississippi. Its present

condition shows the problem in its fullest force. Levees are broken, towns are inundated, miles of country under water, hundreds of families homeless, many lives lost, and all destructible property over an area of hundreds of miles destroyed. This is a condition likely to occur any year, and it must keep a large section of the most productive land in the United States from being developed, so long as it remains unremedied. Few people will care to settle in a territory where they are liable to be drowned and certain to have their stock and dwellings washed away whenever there is an extraordinary rise in the river. It is a problem not easily solved, for even the expensive operation of building levees affords only temporary relief. This plan, pursued on the Yellow River of China, has had the effect of raising the level of the river above that of the surrounding country, so that when the levees break, as they do every once in a while, a vast tract is inundated and the destruction of life and property is enormous. Can American ingenuity find any better scheme for curbing the Mississippi?"

This prediction was verified a few years after, for, on October 15, 1901, it was officially reported that the floods of the Yangtse Valley had reached their highest stage in fifty years, wiping out villages, destroying crops and drowning thousands of people. Despatches just received from the Kwang-Si district state that 150,000 persons are in danger of starvation, and that women and children are being sold to procure food.

In our own country similar catastrophes have been recorded, beginning with the melting of the snows in the spring, causing the early floods in the Ohio, which have formed numerous crevasses in the levees and a higher stage in the Mississippi River than has ever before been recorded. The stories of suffering, devastation, heroic rescues and deaths have been so constantly presented as to appeal earnestly to the sympathies of the more fortunate sections of the country, and no sooner had the waters subsided than a similar disaster was inflicted upon Topeka, Kansas City, St. Louis and other points from the discharge of the western affluents of the basin. At the same time that these north-

ern sections were suffering from an excess of water, the southern portion of Louisiana was praying for rain to save the rice, cotton and sugar crops from destruction by drought in consequence of the exclusion of the water from the land. What is needed, therefore, is a more equable distribution of water for fertilization and a more uniform stage in the river for navigation. The problem is as old as the world and has been successfully applied in the fertile river bottoms of Mesopotamia and Egypt, which for thousands of years were the principal sources of supply of food to mankind.

In this country, however, the present practice seems to be based not so much on experience in olden times as upon experimentation to meet existing conditions, and in commenting upon the results secured to date, the New York *American* says: "The country is stunned by the appalling horror of the calamity in North Topeka, where fire and flood have combined to slay hundreds. It seems incredible . . . and that such a disaster should have been possible is an arraignment of the engineering skill and foresight of our time.

"The world should by this time have reached a stage where the safety of cities from river floods should be a certainty. . . . Because we have not progressed to this attainable stage, Kansas, Nebraska, Missouri and Iowa are to-day mourning their dead."

In behalf of the engineering profession, it seems necessary to state that the responsibility for these disasters does not rest with it, since Congress has the sole jurisdiction and control of these questions, and its attention has frequently been called by engineers and river men to remedial measures which have not been adopted. The real causes may better be ascribed to politics and local interests, as may be illustrated by the following reported interview with one of the most sagacious railroad magnates of this country, now deceased, who, on being requested to lend his aid to the passage of an Act which would have provided earlier relief to the Mississippi Valley, is said to have replied:

"I have read all of the arguments before Congress. The

outlet system is correct and the only way to deepen the Mississippi River. If that were done, what would become of not only my own, but all other railroads coming to New York? They would be ruined, and for that reason he would favor the levee system, for so long as that be kept up, there need be no fears of the Mississippi as a competitor."

A Chicago editor added, whether pertinently or impertinently: "The New Yorkers do not intend to let anything be done that does not pay tribute to New York."

Alas for the rarity of human charity.

It is an unfortunate fact that sectionalism permeates so many of the measures presented to Congress and militates against the general development of the entire country. In this particular instance, however, there is a natural belief that the best means of protecting property from inundation, is to surround it by an earthen dam or dike, so that the earlier settlers in selecting the choice locations for their farms upon lands subject to inundation, were compelled to resort to levees as the simplest and cheapest device for self-preservation. This manifest remedy being innate with all riparian owners, has caused a rapid extension of such protective works as the settlements along the river increased, so that by 1858 it was reported that there was practically a continuous line of levees from Cape Girardeau, Mo., to New Orleans, having an average height of 4 feet. This system, however, proved wholly insufficient for the flood of that year which lasted 199 days, the longest time on record, and covered the whole valley for distances of from 50 to 100 miles to depths averaging 12 feet. In 1823 the levees were only a foot or so high; in 1851 they were raised 5 feet high; in 1859 they were raised 7 feet high, and have been raised and enlarged to meet the requirements of each succeeding high flood which overtopped them until they now average nearly 20 feet.

A Board of U. S. Engineers reported in 1875 that the breaks of the previous year were too numerous to mention. Their total width was 130 miles, and it was added that if the levees do not break the water will run over the tops of them. "The levees are under grade from 6 to 7 feet from

Cairo to New Orleans." This was after 4 feet had been added to the levees of 1858. In 1882 the break at Illaworra, La., in a levee 20 feet high caused damages to the extent of $23,000,000, and it was stated that two or three such breaks will overflow more country than thirty when the levees were from 4 to 6 feet high.*

With a firm faith in the efficacy of an earthen dam to protect and reclaim their farms, and with no knowledge of the various other resources available for the preservation of their lives and property, the riparian dwellers along the lower river are urgent advocates of the continuation of the levee system and instruct their representatives to vote accordingly. Hence, it is that no other systematic effort has been made to modify or improve the conditions inherent to this perennial system of defence—a system which from the very nature of the forces to which it is exposed can never be regarded as complete because of the constantly caving banks which carry with them their superimposed levees.

The efforts to confine the river to a fixed channel by revetting its banks has been discontinued because of the enormous expense and unsatisfactory results, and recourse is had to dredging for relief at low water. But this is not always available, as is instanced by the recent imprisonment of the Monitor Arkansas, near St. Genevieve, Mo., just after the early spring flood had passed out, when it was expected that she might be thrown out of commission for many months or until dredges could be used to open an 11-foot channel to deep water. The Missouri flood, however, released her before the dredges were available.

Physical Elements.—It is a fundamental principle of hydromechanics that *if the capacity for outflow be made greater than that for inflow there can be no overflow*, and yet there are intelligent advocates of levees who seem to be so wedded to the idea that all the water must be confined to the trunk of the river, or be under the control of levees along the banks, that they deny the applicability of the principle to this stream and insist that the more water the more

* See Document No. 123 Senate, Fifty-fifth Congress, First Session, 1897.

the velocity, and the more the work of scour. But it should be added that if the water is poured in at the upper end faster than it can be voided at the mouth, it must be dammed back and its velocity be retarded by the impounding of the volume, and hence the sediment will be dropped in the stream and the crossing bars be raised because of the higher stages of the flood. This is abundantly proven on a large scale at the South Pass, where the two jetties (levees) retard the flow and have filled up along the entire reach of the river above for 12 miles, so that where at one

MAP OF THE MISSISSIPPI DRAINAGE SYSTEM

FIG. 1.

time the depths were over 70 feet, they are now less than 30, and yet it is asserted that similar congestions in the main trunk do not cause the bed to rise, because careful surveys made at a few years' interval appear to show no such deposits. The question of the rate of sedimentation will be considered later if time permits.

Physical.—To understand the problem fully it is important to look briefly at some of the elements which affect the movements in the drainage basin, taken as a whole. These are its form, area, relief, slopes, rainfall, runoff, vegetation,

temperatures, etc., most of which may be concisely presented in the form of a tabular statement, as follows:

PHYSICS OF DRAINAGE BASINS OF MISSISSIPPI VALLEY.

Name.	Area.	Annual Rain.	Total c.f. of Precipitation.	Run Off, c.f.	Mean Disc.	Ratio.	Elevation at Source.	Length.	Elevation at	
Name	Sq. mls.	in.			per c.f.		feet.	miles	feet	miles
Up. Miss. . .	169,000	35·2	13,800,000,000,000	3,300,000,000,000	105,000	0·24	1,689	1,330	St. Paul, 670	658
Missouri . .	518,000	20·9	25,200,000,000,000	3,780,000,000,000	120,000	0·15	6,800	2,908	Mouth, 281	—
Ohio R. . .	214,000	41·5	23,700,000,000,000	5,000,000,000,000	158,000	0·24	1,649	1,265	Pitts., 699 Mouth, 275	975
Arkansas . .	189,000	29·3	13,000,000,000,000	2,000,000,000,000	63,000	0·15	10,000	1,514	Mouth, 162	—
Red	97,000	39·0	8,600,000,000,000	1,800,000,000,000	57,000	0·20	2,450	1,210	Mouth, 54	—
Yazoo . . .	13,000	46·3	1,500,000,000,000	1,300,000,000,000	43,000	0·90	210	500	Mouth, 103	—
St. Francis and others	10,500	41.1	1,100,000,000,000	990,000,000,000	31,000	0·90	1,150	380	Mouth, 200	—
Small tributaries	32,400	—	3,600,000,000,000	3,240,000,000,000	—	—	—	—	—	—
Lower Miss.	—	—	—	—	675,000	0·25	416*	1,286*	Mouth, 0	—
	1,242,900		87,700,000,000,000	21,410,000,000,000						

* High water at mouth of Missouri River.

In connection with this table it is well to give the areas of the natural receiving basins which are subject to inundation by the unrestrained waters overflowing their banks:

AREAS OF NATURAL SEDIMENT BASINS.

	Sq. Miles.
St. Francis, on west bank	6,706
White River " " "	956
Tensas " " "	5,370
Atchafalaya " " "	8,109
Yazoo, on east bank	6,648
Ponchartrain, on east bank	2,001

Total area, 29,790 square miles, which is about 0·024 per cent. of the entire basin. If the entire discharge be confined between levees from Cairo to the mouth of the Red River, where the outlets begin, the area would be reduced to only about 4,500 square miles, or 15 per cent., thus leaving say 85 per cent. from which the river water is intended to

be excluded and the natural process of fertilization and sedimentation is cut off. As a large portion of this is swamp land which cannot be drained, because of its level being below that of the river, it would seem to be unwise thus to deflect the only means by which it might have been gradually elevated and made habitable. Moreover, the area included between the existing levees is far from being that ideal trough which should be provided for the maintenance of a uniform flow, so necessary to the creation of a navigable channel, as will presently appear.

From the above table it will be seen that if the floods from all the tributaries were simultaneous the lower river channel between levees would be entirely inadequate and the country would be uninhabitable; but, fortunately, in the Divine economy, the rigid hand of the Ice King seals up the more remote and higher sources of the western floods until those from the Appalachian chain have in large part passed down to the sea.

It is a well-known fact that the slopes of the streams diminish as they approach their mouths and that the sedi ment which they transport grows smaller in size. It must, therefore, follow that the coarser gravels and shingle deposited in the higher reaches cannot be carried throughout the entire length of the stream and be dumped into the Gulf. It must, consequently, remain in the bed of the river, causing it to rise; and yet it is claimed that such is not the case, for if the bed rises the height of the floods must increase proportionately and the levees also. Upon this denial of bed elevation depends the argument in support of levees, whose advocates maintain that they control the floods and cause compensation for the increased volume by deepening. Careful surveys have been made over long stretches of the trunk of the river, but it is conceded that they prove nothing, while the increasing height of the floods and levees point strongly to a confirmation of the opinion that the bed is silting up. It certainly seems rational that if there is an increasing scour in the higher reaches, that the lesser slopes at the mouth, which are at times negative, would arrest the movement and thus obstruct the dis-

charge and raise the flood-plane. This view is confirmed by the composite cross-sections just issued by the Mississippi River Commission, showing deposit in the low-water areas and caving of banks above that stage.

Rate of Sedimentation.—It has been said by them of old time, that "All the rivers run to the sea, yet the sea is not full; unto the place from whence the rivers come, thither they return again." While this is true of the waters of the rivers it is not true of the sediment which they carry. The millions of tons of rain, aided by the frost and winds, denude the mountain fastness, so rich in mineral and fertilizing elements, and feed it to the great hydraulic conveyors which transport it to and deposit it upon these garden spots of the earth, built up of the material supplied by the mountains.

It is stated that since the mouth of the river was at Commerce, just above the Ohio, the deposits have covered some 40,000 square miles, which have been reclaimed from the Gulf and elevated to a height of 285 feet at Cairo, sloping both laterally and longitudinally, so that the river runs on the ridge which it has built up from the bed of the Gulf.

This feature of alluvial rivers is manifest by a glance at the delta, which so clearly exhibits the process of land structure by the long, tapering salients which carry the several arms of the stream far into the Gulf, instead of permitting it to escape down the shortest and steepest slope to the level of the sea. A cross-section of the river also shows the elevation of the banks to be highest along the borders of the stream, and that it falls away in the distance at the average rate of about 3 or 4 feet in each mile, so that the overflow drainage is away from the river and into the bayous and swamps which are its natural sedimentation basins.

It has been estimated that the volume of sediment carried into the Gulf each year varies from 250,000,000 to 278,000,000 cubic yards, but it is also stated that the banks are constantly caving at the rate of about 9 acres per mile of river and that their average height is from 30 to 45 feet, which would contribute about 332,000,000 cubic yards to the bed

of the stream, independently of the amount fed by the tributaries, which is no doubt much more, so that the net amount of sediment left in the bed would exceed that deposited in the Gulf, if it be prevented from spreading beyond the bank by levees. Hence it must follow that unless the ejecta are equal to the injecta the excess must contribute to elevation. This may readily be accepted as a fact when it is seen that a single flood may cause deposits on a level amounting to from 6 to 12 inches of mud.

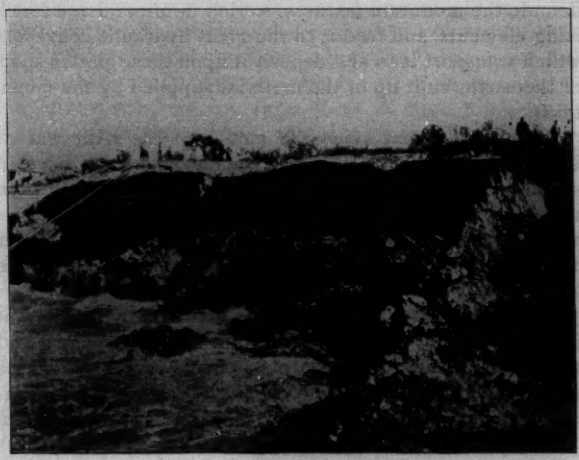


FIG. 2.—Banks in the act of caving.

It is well known that any alternations of velocity will cause corresponding scour or deposit, and that such alternations are caused by variations of cross-section such as are produced by the great fluctuations in the distances between the levees and banks, ranging, as they do, from less than 2 miles to more than 20, so that large areas of deposit are created in the bed of the river, covering in some cases more than 100 square miles.

(See the chart of the Third Engineer District.)

HYDROMECHANICS.—OBSTACLES TO DISCHARGE DURING FLOODS.

Benefit of Outlets.—To dam a river, in whole or in part, restricts its flow and invites deposits. It is immaterial whether the obstruction be composed of stone, wood, earth, silt, water or air; or what its form or position, so long as it encroaches upon the natural waterway.

During the June flood at St. Louis, after the river had begun to fall, the wind veered and raised it over half a foot, causing a breach in the banks and a large additional loss of property.

In 1860 it raised the level of the river near New Orleans 4·6 feet in two hours, inflicting great damage. The level of Lake Borgne was raised 11 feet at the same time. On September 8, 1900, a hurricane swept over the city of Galveston, raising the height of the water 16 feet above the mean Gulf level, and destroyed more than 5,000 lives. Numerous instances might be cited where the winds have operated to impound waters and cause loss of life and property. In a similar manner the waters of a tributary stream in flood have checked those of the main trunk, or the reverse, and caused overflows and consequent loss. Two or more tributaries discharging simultaneously above an engorged reach, must also result in an overflow. These contractions, which are so numerous along the lower Mississippi, operate to retard the flow above them, causing deposits, not scour. It is mainly when the waters of a basin are permitted to discharge freely into a lower level, or under a "head," that scour takes place, hence the use of tidal sluicing basins in many countries to maintain short channels over ocean bars, by flushing at low water. Hence, also, the scouring effects observable in consequence of crevasses in levees, which instead of injuring the lower reaches of the main river are found to improve them.

The Gulf itself is a water dam which, by its inertia, checks the discharge of the river and deposits its silt across the path of the currents, and no amount of jetty extension can change the operation of this law. Moreover, the salt water being heavier than the fresh, buoys the latter up and

causes it to float seaward on an ascending gradient so that the limit of the depth of the scour of fresh water is restricted to about 12 feet at a distance of 4 miles from the bank. Temporary modifications may be effected by rapid extensions of the bank, in the form of jetties, but these contractions merely increase the resistance to the discharge, and by raising the flood plane cause new breaches in the low banks above, as at "The Jump and Cubitts Gap." They do not, therefore, relieve the river, but invite further disaster and new outlets.

These commonplace suggestions are prompted by the oft repeated remarks that "the greatest good will be derived from the complete control of the floods between levees," which, it is added, "involves no dangers," and that "it is an error to allow large quantities to escape, hence the outlet propaganda is a 'plague' and is condemned by such high authorities as Generals Humphreys and Abbot."

It may be interesting to state the correct report of these eminent authorities in this connection, as it has been so erroneously quoted and misused to sustain the idea that levees are the sole protection from floods. They say, on page 394, "Physics and Hydraulics of the Mississippi:"

"The facts above cited establish that there is no evidence that any filling up of the river bed ever did occur in consequence of a high-water outlet; and, moreover, that it is impossible that it ever should occur, either from the deposition of sedimentary matter held in suspension or from the accumulation of material drifting along the bottom. The conclusion is then inevitable, that *so far as the river itself is concerned, they *are of great utility.* Few practical problems admit of so positive a solution."

In its application, however, they found some practical difficulties from the fear that Lake Ponchartrain would be filled up in a long period of time. These views as to the utility of outlets are fully sustained by the description of the outlet reach of the river extending from the mouth of the Red River to the Gulf, as reported by a member of the

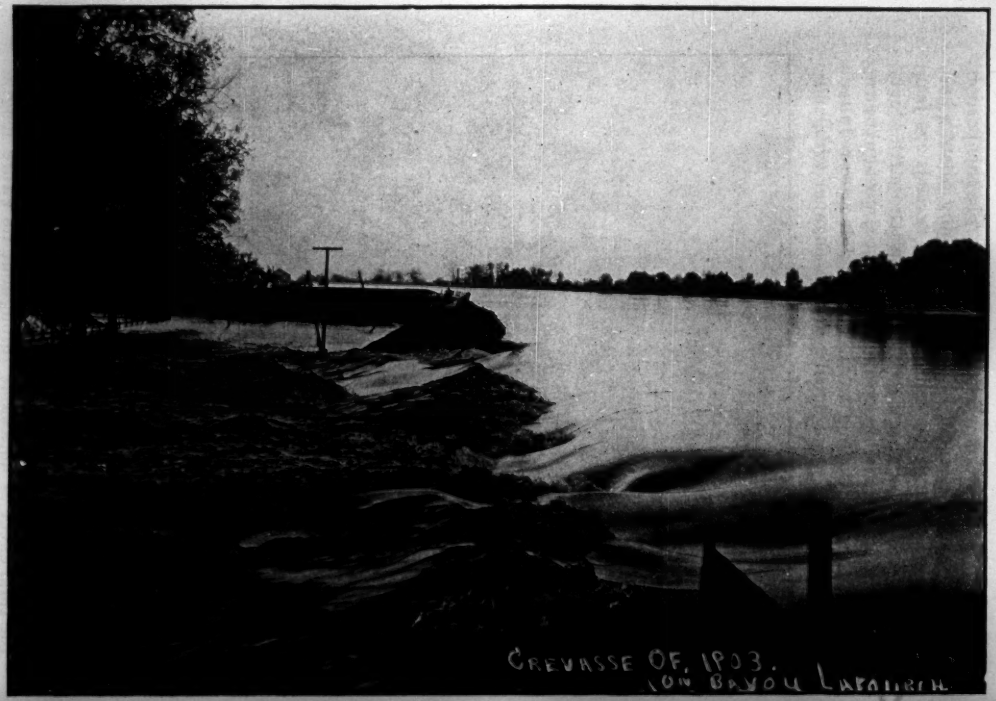
* Outlets.

Mississippi River Commission, who says: "In this portion of the river the channel is narrow, averaging about a half mile in width and the depth sometimes exceeds 200 feet. Bank erosion is slight as compared with the fourth reach, sand bars as obstructions to navigation are almost unknown, and neither contraction works nor dredging are required. Here, nature has constructed an ideal channel with suffi-

FIG. 3.—The outlet section from Red River to the Gulf, showing parallelism of the levees.

cient depths at all times for the largest sea-going craft. For nearly the entire length the water is confined to a single channel down to the Head of the Passes, there being but two island, Profit and Bayou Goula, in the entire reach.

"As the upper limit of the reach at low water is less than 2 feet above mean Gulf level, it often happens that the tidal effect is noticeable throughout its entire length.


CREVASSE OF 1903.
ON BAYOU LAFOURCHE

The Red River is the only tributary in the fifth reach which adds to the volume of the main stream.

"There are several outlets through which the waters of the river can flow to the Gulf of Mexico. Some have been closed by levees, as Manchac and Plaquemine Bayous. The Achafalaya, La Fourche, Collet, The Jump and Cubitts are still open, but are gradually filling up with sediments. . . The extreme oscillation in stage recorded up to the present time at New Orleans is 20·7 feet."

Thus nature has demonstrated the great superiority of this stretch of river where the fluctuations are less than one half those of the higher reaches, the channel far more permanent, the banks stable and the protection far superior, because of the safety-valves afforded by the various outlets, which unfortunately are slowly filling as the sediment is left in them from the overflowing waters, thus raising their beds and reducing their capacity. The effect of this congestion was forcibly illustrated by the crevasses which occurred this year on the Bayou La Fourche, in front of Hon. W. E. Howell's plantation, as shown in the accompanying illustration, taken from the *Louisiana Planter* of June 20, 1903. If man does not provide the necessary outlets for the rapid discharge of the floods, nature seeks relief where it is least expected and without warning, carrying ruin in her wake.

The increase in stage due to the extension and elevation of levees by attempting to "control" the entire floods is shown forcibly from the gauge record at Carrolton since 1828, when it registered 14·85 feet; in 1851, it was 15·05; in 1874, 15·35; in 1897, 18·72 and in 1903, 19·05.

Serious Defects of the Levees.—From the very crude manner in which the levees were developed, by localities and individual planters, it was not practicable at first to construct such a system as is desired to train the river in its onward course to the sea. These fragmentary low levees were gradually connected and built up along former lines until there resulted an angular alignment of great variability of cross-section between banks or bluffs causing serious contractions to throttle the floods and impound the high

waters in the bed of the stream, thus creating pools and chutes which are so much to be avoided in the maintenance of a navigable channel. For example, the reach extending from the mouth of the Arkansas River to Vicksburg, at the mouth of the Yazoo basin, may be taken as a type. Here it is seen that at the outlet of the Arkansas the distance between banks is about 17 miles, while at Arkansas City it is contracted to less than 2 miles by a salient line of levees which might apparently have been located a few miles farther back, thus saving about 5 miles of levee work and affording a much better discharge. Again, at Greenville a pool exists 8 miles wide by about 12 long at high water, with a similar gorge below it which has caused serious crevasses in this section. So throughout the 213 miles in this district, these features recur at frequent intervals, the worst being at Vicksburg, where the gorge creates the backwater which inundates the lower Yazoo district. It is evident that the relief must come, not by beginning the rectification of the levee alignment at the top, thus pouring more flood water into the lower pools, but by opening the throats of these congested sections from the mouth up and letting the water out more rapidly, thus creating a scour which will deepen the bed and lower the flood, as in numerous foreign rivers. This would reduce the cost of maintaining the levees by diminishing their length and give a better channel with less cost for dredging. The land thus forfeited would be more than gained by the immunity from inundation of the back country and the drainage of other territory.

Discharge Capacity.—To determine the effect of bends upon the discharge of a conduit, some experiments have been made by the writer on tubes of different diameters, two of which represented the low-water channel, one being flexed to conform to its convolutions and the other straight to represent the developed equivalent length, and the time required to pass a given volume of water through them, under a constant head, was noted. From these experiments it was found that the bent tube had a capacity of only 17 per cent. of its equivalent straight tube. Similar tests on the two larger, high-water tubes showed that the one repre-

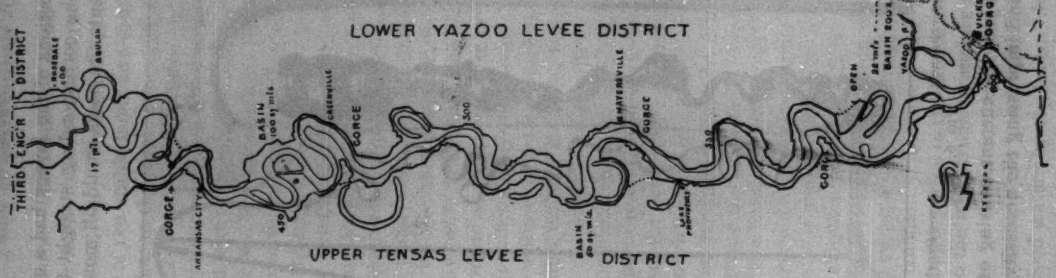
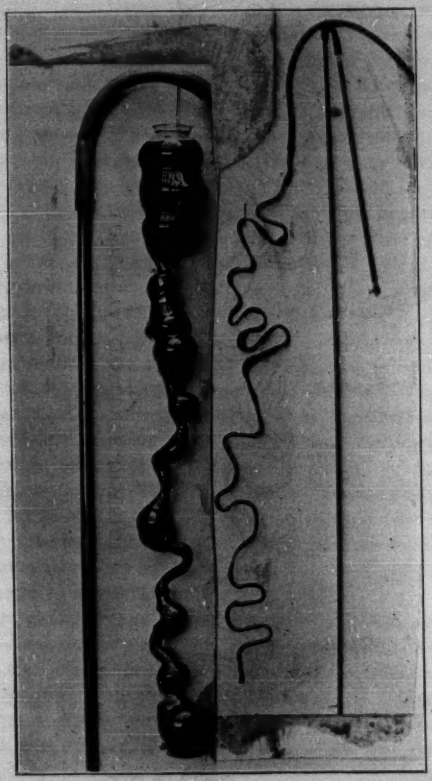

LOWER YAZOO LEVEE DISTRICT
THIRD ENG'R DISTRICT
GREENVILLE
GORGE
VICKSBURG
GORGE
OPEN
UPPER TENSAS LEVEE DISTRICT

FIG. 5.

senting the distortions of the levees had a capacity of but 44 per cent. of the corresponding straight tube. In this series of experiments the pools would fill more rapidly than



High-water channels. Low-water channels.

FIG. 6.

the gorges could pass the current, causing overflows, as in a state of nature, and impressing the necessity of a uniformly canalized channel.

Relief Measures.—The resultant of all the experience of the works done on the river shows that there are two distinct problems to be solved. The one relates to the reclamation and protection of the arable lands as a source of revenue and supply, the other to the improvement of the channel of the river as a means of cheap distribution; and the methods now applied to meet these requirements are the attempt to confine the entire flood to the irregular sections between high levees and to improve the low-water stages by the removal of the bars built by the floods, by dredging channels, which are obliterated at the next high water.

Under these circumstances it would seem that the problem is still an open one and that there is room for improvement, and although the engineering profession has no direct control, it may perhaps make suggestions in the interests of humanity and for the betterment of the channel, which, if carried out, may result in less loss and risk to life and property. These remedial measures must evidently be directed towards the prevention of simultaneous floods and accumulation of large volumes of water, which must be carried in a congested conduit for long distances before it can find relief, and therefore no single system will fulfil these requirements but a combination of impounding reservoirs on the tributaries, especially of the western sections, where they may be utilized for irrigation; reforestation of the arid regions; levees with a readjusted alignment and low-weirs at suitable points for spill-ways, to be connected with large subsiding reservoirs in the swampy bayous; and, finally, by the removal of the bars at the delta and opening of all the available and possible outlets to permit of a rapid voiding of the excess of the floods. By such a combination it is believed the navigable channel will be deepened and be made more permanent, the flood plane will be lowered and less arable land be inundated, the height of the levees will not have to be constantly increased, the discharge will be retained for a longer period in the lateral basins, from which it may be fed to the main trunk to sustain a nearly uniform stage and water be stored for irrigation, which is already becoming a necessity in the lower valley because of the exclusion of the flood waters by the levees.

The practicability of the lateral basin system as an auxiliary has been carefully considered and ably presented by James A. Seddon, C. E., for years engaged on the hydraulics of the Missouri and Mississippi rivers. He has shown, by a series of hydrographs, the possibility of so regulating the flow of the river as to reduce the flood heights from its present extreme range of 50 feet to about 25 feet, by the construction of six reservoirs along the back-waters of the St. Francis basin, where the fall is about 120 feet from Cairo to Helena. This work is estimated to cost $32,000,000, including land damages and auxiliary works, and can be built in ten years.

The time is too limited to do this subject justice, but in view of the impending danger from the augmented volume, it is never too late to call attention to the possibilities of relief, which is all that the engineering profession can do until the people of the valley awaken to the necessity of their application and demand a change of plan.

SCARCITY OF LUMBER.

Recent forest fires in the timber belt of Maine have devastated large tracts of land, hundreds of square miles, which cannot be replaced in many years, and the freshets in the Mississippi have carried away millions of feet of logs, which will never be recovered. A writer in the *Sun* says that more lumber has been destroyed this season by misadventure than manufacturers can cut in a year. These casualties and the natural consumption tend to make commercial timber scarce and high, and those who contemplate building where wood is required are advised to do so immediately, as it will be a long time before it can again be had at present rates. Southern lumbermen who furnish yellow pine are ninety days behind their orders, and the price has been advanced on some grades $2 per 1,000 feet. The writer above quoted says that forestry is a very slow remedy for renewal of standing wood, for it takes thirty to forty years to grow trees to 24 inches diameter, which is the size demanded for boards. The supply of hemlock in the immediate vicinity of this market has been exhausted, and people who require it are compelled to buy 300 miles away. There is practically no wood of this kind left in Pennsylvania, unless it may be in the western part of the State, and when this is cut and consumed it will be necessary to use other varieties. White pine, oak and cypress have advanced during the year from $6 to $14 per 1,000 feet. The increase in population annually reaches large figures, and the consumption of lumber must keep pace with it, but from the facts quoted there will be a shrinkage instead of a surplus over former years. In this condition of affairs the importation of foreign woods from our new possessions should receive a decided impulse.

The Induction Motor and Its Engineering Capabilities.*

[*A Thesis Presented to the University Faculty of Cornell University for the Degree of Doctor of Philosophy.*]

By George L. Hoxie.

(*Continued from p. 214.*)

Part II.

Operation of the Motor.

Qualitative.—We have seen that the primary currents produce a regular magnetic field, and that when the secondary carries no current, sensibly all of this field enters the secondary structure, and surrounds the secondary conductors.

If the secondary be stationary, an E. M. F. will be induced in each secondary bar, that is practically equal to the counter E.M.F. which the same flux produces in each primary conductor. Let the secondary bars be joined together at each end of the motor by rings of conducting material, but having a very high resistance. Since there are six poles with flux entering, and six with flux leaving, the secondary, there will be six regions on the rotor where secondary current will flow in one direction, and six regions where the currents flow in the opposite direction.

The developed diagram for the secondary current circuits is found in *Fig. 14.* The ladder-shaped figure represents thirty-five of the secondary bars, and the end rings which complete the circuits, and make possible the flow of current. It will be remembered that the E.M.F. induced in a single primary conductor was such as would be produced by a sine wave of flux, crossing it with a uniform velocity. If the total flux crosses into the secondary, an equal E.M.F. will be induced in each secondary conductor, as long as it remains stationary. Above the ladder in *Fig. 14* is drawn a sine wave which may represent an equivalent

flux wave for this motor. It is drawn in the proper relation to the secondary conductors. A uniform progression of such a wave would generate E.M.F.'s in the secondary conductors, the amount and direction of which are represented by the arrows of *Fig. 14*.

We have assumed that the end rings joining the secondary conductors together, have a very high resistance. Let us further assume (incorrectly) that this resistance is the same for each current path. Then small currents will flow in the secondary, and these currents will be in phase with, and proportional to, the E.M.F.'s which establish them,

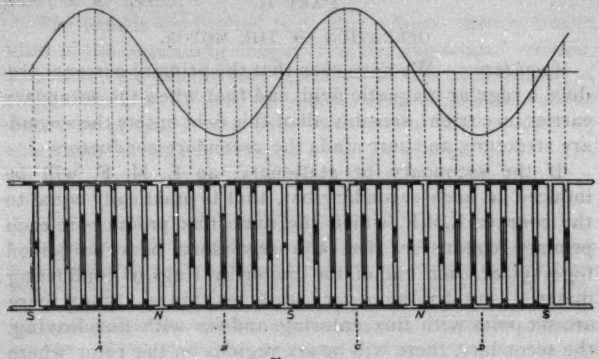


FIG. 14.

and the currents and E.M.F.'s may be represented by the same arrows.

The division of the currents into current groups, or circuits, is indicated by the dotted lines *A*, *B*, *C*, and *D*, in *Fig. 14*. If such current circuits were present alone in the machine, they would produce magnetic circuits, linked with themselves, and having maximum densities at the points *N*, *S*, etc., of the figure. That is, the secondary currents would tend to set up a field 90° behind the field due to the primary. The distribution of such a field would have a sine form. It could only be small as compared with the primary field, since we have assumed a large secondary

resistance, and a small current, in order that current and E.M.F. may be in phase.

The combination of secondary and primary fields would give a resultant field slightly shifted back from the position that we have assumed, and would introduce some minor changes in the above discussion, which we are not yet ready to consider.

Torque.—It may be noticed in *Fig. 14* that the maximum currents lie directly under the maximum generating E.M.F.'s. If the conductors lay on the surface of the secondary, they would experience a force urging them from right to left. All the forces present would be in the same direction, and the secondary structure, if free to do so, would revolve.

In the actual motor, the secondary conductors lie in slots and probably experience little or no force. The resultant effect is, however, quite the same, since the actual mechanical force is in any case due to the tension of flux which is distorted by the current. This distortion, in the case of embedded conductors, is still produced in about the same way, but the tension is transferred to the secondary teeth, since the flux mainly enters them.

Secondary With Little Resistance.—Quite a different state of affairs is found if the secondary conducting circuits contain little resistance. Imagine the secondary to be stationary. The secondary E.M.F. is produced as before. The currents flowing are mainly limited by self-induction, and will therefore lag nearly 90° behind the E.M.F.'s producing them, or will lie 90° to the right of the E.M.F. positions of *Fig. 14*. This brings the M.M.F. of the secondary directly opposed to the M.M.F. of the primary. The flux surrounding the primary conductors must remain nearly the same. The primary current must therefore increase greatly,—so greatly that a considerable part of the primary flux, prevented from entering the secondary, leaks along the air gap, and crosses between the primary teeth near their ends.

The torque present is nearly as much in one direction as the other, and the net turning force on the secondary structure is very small.

Moderate Resistance in End Rings.—As these motors are usually constructed, the end rings are composed of an alloy having a considerably higher resistance than copper. The secondary currents, with the motor stationary, are partly limited by self-induction, and partly by resistance. The secondary currents lag behind the E.M.F.'s that produce them, but the lag is considerably less than 90°. (The exact angle of lag depends of course upon the ratio of self-inductive reactance to resistance.) The torque is considerable, and the primary current is smaller than without the high resistance end rings. Since some current paths include more of the end rings than others, the currents are no longer proportional to the E.M.F.'s, and the secondary flux wave taken alone is somewhat more pointed than a sine wave.

Secondary Revolving.—Since the pull on the secondary, or rotor, is in the direction in which the magnetic field travels, its rotation will be in the same direction. In general the rotor travels somewhat slower than the flux. This means that the relative speed of the flux, referred to the secondary conductors, is decreased, and the induced E.M.F., and the frequency, is less. The secondary resistance is unchanged, but its inductive reactance, which is proportional to the frequency, is lessened. The secondary currents are in a better position for the production of torque, and are less in opposition to the primary currents, when the motor is running than when it is standing still. The magnetic leakage for a given secondary current is considerably lessened for the same reason.

Slip.—The field, or the equivalent field, due to the primary currents, revolves at a rate dependent upon the frequency of the primary impressed pressure.

If the rotor is stationary, the secondary frequency is equal to the primary frequency. If the rotor is revolving at the same rate as the flux, *i. e.*, at "synchronous speed," the secondary frequency is zero. The speed of the rotor is frequently measured in "per cent. of synchronism." The difference between this number and 100 is known as the "per cent. of slip," or the "slip." The secondary frequency is the product of this slip, and the primary frequency.

Relations of Torque, Current and Speed.—If no self-induction is present in the secondary—a case which may be represented by *Fig. 14*—the torque depends solely upon the secondary current, and therefore upon the secondary E.M.F., or inversely upon the slip, no leakage being present when self-induction is absent. Such a motor would be a practical impossibility, but would have a straight torque speed characteristic, running from zero torque and synchronous speed, to maximum torque, and zero speed. The power-speed curve would reach a maximum at half-speed, and would be zero at synchronous speed, and at zero speed, the two sides of this curve being symmetrical.

It is of course impossible to approximate, in the actual motor, to a condition of no self-induction, or even to a very small self-induction. It is possible, however, to accomplish about the same results, by increasing the secondary resistance, thereby making its self-induction relatively less important. Such a motor will have a torque decreasing with an increase in speed, or conversely, the motor will vary in speed with varying demands upon it for torque. In other words, it is to some extent a variable speed motor. Owing to the secondary resistance, considerable amounts of energy are lost in heating the secondary conductors and the efficiency of the motor is thereby lowered.

If the resistance of a motor secondary is very low, its inductive reactance is large in comparison, except for speeds so near synchronism that the secondary frequency is only a small fraction of that of the primary. At all other speeds the secondary currents lag greatly behind the induced secondary E.M.F.'s, and, as previously shown, the torque is small. At the speeds near synchronism the slip is small, and the secondary E.M.F. is small, but as the resistance is low, considerable secondary currents may be present. Such a motor will exert any torque between zero and its maximum, with a decrease in speed of only a few per cent. It is therefore practically a constant speed motor.

The starting torque of the variable speed motor is high, while that of the constant speed motor is low. The secondary current of the former varies with the torque. The

secondary current of the latter varies with the torque, and power, up to about maximum torque and remains fairly constant after that, while the speed of the motor is reduced.

Primary Leakage Wave.—The primary leakage flux must travel along the air gap through a region of high reluctance, and practically constant permeability. The paths of this flux should extend from tooth to tooth, as for the useful flux, *i. e.*, in *Fig. 7*, leakage would be found between teeth 6–8 : 5–9 : 4–10, etc. Calling a distance along the air gap occupied by one tooth and one slot as representing one unit of reluctance, we would have in the motor previously studied :

Path No.	Reluctance units.
1	12
2	10
3	8
4	6
5	4
6	2

Fig. 15 gives the corresponding leakage waves. They are derived from *Fig. 4*, by dividing each ordinate by the corresponding reluctance unit. It should be noted that the primary leakage flux is in a constant phase position with respect to the primary current. That is, its position is not altered by the secondary currents, and the primary impressed pressure that is consumed by the leakage counter E.M.F., is always 90° in advance of the primary current. This means that a large primary leakage is accompanied by a low primary power factor.

Primary Power Factor.—When no secondary current flows, the useful primary flux is in phase with the leakage flux, or the primary counter E.M.F. is 90° behind the primary current. The primary impressed E.M.F. is mainly made up of a component balancing the counter E.M.F., and is therefore almost 90° ahead of the primary current. The power factor in this case is very low. Energy is consumed by primary resistance, and by hysteresis losses, and by eddy currents in the iron only.

In the discussion of *Fig. 14*, it was shown that the first small secondary currents set up a field 90° behind the primary

field. If the two were of equal strength, the resultant would be 45° behind the *original* primary field. But as it is the *resultant* field which sets up the secondary E.M.F., a resultant field lagging 45° would give secondary E.M.F.'s (or currents, if the secondary were without self-induction) that would be 90° plus 45°, or 135° behind the original primary field, and this in turn would shift the secondary current again, and so on till a balance was reached at a point depending on the relative values of primary and secondary currents. The final resultant field would obviously lag behind the position of the original primary field, but the lag could never be as great as 90°. If the lag were 90°, the counter E.M.F. in the

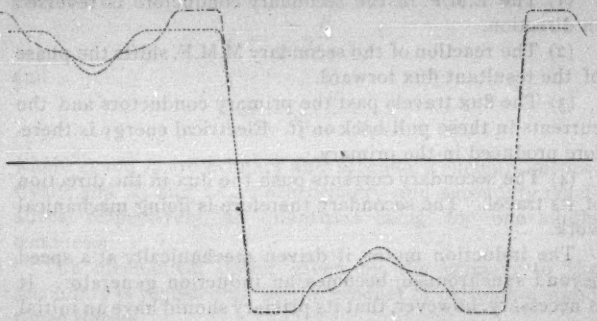


FIG. 15.

primary would be 180° behind the current, or the impressed E.M.F. would be in phase with the primary current—a condition where the power factor in the primary would be unity.

Transfer of Power Between Primary and Secondary.—The power given to the primary is transmitted to the secondary through the medium of the resultant magnetic field. The primary currents are pulling this field forward, and the secondary currents are pulling it backward, while the field itself is changing in a manner which amounts in effect to a continuous forward motion. The primary is therefore doing work, and the secondary is receiving it, or rather is passing it along as mechanical power.

When the secondary has self-induction, the secondary currents may lag up to something less than 90° behind the induced secondary E.M.F. The secondary M.M.F. may therefore be brought nearly into opposition with the resultant flux. This, as we have already seen, does not involve a transfer of power, nor does it tend to shift the resultant flux from the phase position of the primary. A large primary current is drawn at a very low power factor.

Mechanically Driven Secondary.—If the secondary be driven in the direction of normal rotation, at a speed faster than the equivalent rotating flux wave, a rather peculiar state of affairs results:

(1) The E.M.F. in the secondary conductors is reversed in direction.

(2) The reaction of the secondary M.M.F. shifts the phase of the resultant flux forward.

(3) The flux travels past the primary conductors and the currents in these pull back on it. Electrical energy is therefore produced in the primary.

(4) The secondary currents push the flux in the direction of its travel. The secondary therefore is doing mechanical work.

The induction motor, if driven mechanically at a speed beyond synchronism, becomes an induction generator. It is necessary, however, that its primary should have an initial excitation.

Two-Phase Motor.—When the primary of an induction motor is suitably wound and is excited with a two-phase current, the flux changes which occur in its magnetic circuit are very similar to those that have been studied in connection with the three-phase model. The preceding discussions will, therefore, apply without material change to the case of a two-phase machine.

Induction Motor Equations.—If Φ = flux per primary pole, and Φ_m = flux per pole that is common to primary and secondary, then, if the impressed E.M.F. wave has a sine form, the counter E.M.F. per primary conductor is,

$$E_1 = \frac{2 \cdot \Phi \cdot f}{10^8} \text{ 1·11},$$

and the induced E.M.F. per secondary conductor is,

$$E_2 = \frac{2 \cdot \Phi_m \cdot f}{10^8} 1{\cdot}11 \cdot sl.$$

where (f) is the primary frequency, and (sl) is the slip in per cent. of synchronous speed.

The primary impressed E.M.F. per conductor is given by the expression,

$E = E_1$ (in phase with flux) $+ I_1 R_1$ (in phase with current).

This addition must be geometrical.

The values of primary and secondary current per conductor are,

$$I_1 = \frac{E}{\sqrt{R_1^2 + X_1^2}}$$

and

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + (X_2 sl)^2}}$$

where I_1 and I_2 are primary and secondary currents respectively, and X_1 and $(X_2 sl)$ are primary and secondary reactances, respectively, all quantities being for one single conductor.

The mechanical power given by the primary to the secondary is represented by the product of peripheral speed times torque.

We have seen that the torque is due to a continuous forward pull of the field, the latter having been distorted backward by the secondary currents. This torque for a given conductor is directly proportional to the density of the field and to the amount of the current. Such a force for the given conductor in air would be given by the expression,

$$F = B \cdot l \cdot I',$$

all quantities being in c. g. s. units.

The power expended is proportional to the force, and to the velocity of the conductors. An expression for the power per secondary conductor would therefore be,

$$P = Fv = B \cdot l \cdot v \cdot I'.$$

This is equivalent to writing,

$$P = E'I',$$

where E' is an ideal E. M. F., not actually present in the secondary, but which would be induced were the magnetic field to remain stationary while the secondary revolved with the given speed.

Calling this ideal E.M.F. E_m, since it is analogous to the counter, or motor, E.M.F. which is found in a direct current motor, we may write for the mechanical power given to the secondary,

$$P = E_m I_2 \cos \theta_2,$$

and since $E_m = \frac{2 \cdot \Phi_m \cdot f}{10^8} 1.11 (1 - sl) = \frac{E_2}{sl}(1 - sl)$,

$$P = I_2 \frac{E_2}{sl}(1 - sl) \cos \theta_2, \text{ in watts,}$$

or,
$$= \frac{I_2 \cdot E_2 (1 - sl)}{746 \cdot sl} \cos \theta_2, \text{ in horse-power.}$$

If (n) is the speed in revolutions per minute of the equivalent rotating field, the torque in pounds at one foot radius is,

$$T = \frac{I_2 E_2 (1 - sl)}{746 \cdot sl} \times \frac{3300}{2 \pi n} \cos \theta_2.$$

Or substituting for I_2 its value in terms of secondary resistance, and reactance, per conductor, we have the values,

$$HP = \frac{E_2^2 R_2}{R_2^2 + (X_2 \cdot sl)^2} \times \frac{1 - sl}{sl} \times \frac{1}{746}$$

$$T = \frac{E_2^2 R_2}{R_2^2 + (X_2 \cdot sl)^2} \times \frac{1 - sl}{sl} \times \frac{33000}{746 \cdot 2 \pi n}$$

The two formulæ above may be used in the design of a polyphase induction motor. It may be noted, however, that the formulæ imply a knowledge of X_2, a quantity depending upon secondary leakage, and of E_2, a quantity depending mainly upon primary leakage. The designer of induction motors would depend mainly upon the results of tests of previous machines, for data upon which to base computa-

tions of magnetic leakage. Such data is not made public, but is closely guarded by the manufacturing companies that possess it.

Motor Characteristics.—With known values for X_1, X_2, etc., one may make substitutions in the equations for horse-power and torque, for several different values of slip, and may plot the characteristic curves of the motor.

The manner in which the torque and speed relationships depend upon the ratio of secondary resistance to secondary reactance, is well indicated by the torque-speed curves of

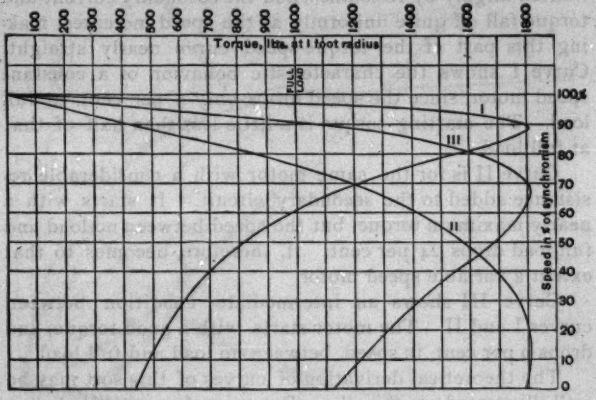


FIG. 16.

Fig. 16. These curves were given by Mr. C. P. Steinmetz (*Trans. A. I. E. E.*, Vol. II, page 35), and are for a three-phase motor, the capacity of which at full load was 100 horse-power.

Curve I is for this motor, with the lowest possible secondary resistance. The ratio of secondary reactance to resistance is large when the motor is stationary, and is zero when the motor is running at synchronous speed. With the motor stationary, therefore, the lag of the secondary current is large, and the motor torque small. As the motor speeds up, the secondary frequency decreases, and the self-

inductive reactance decreases. The secondary currents are not plotted, but since the secondary impedance is mainly reactive, and varies almost directly with the slip, the secondary current will remain quite constant at the lower speeds. The torque, therefore, increases with the speed, to a maximum of 1·8 times the required torque at full load, reaching this maximum value at a slip of about 10 per cent. With higher speeds, a decrease in slip decreases the secondary E.M.F. proportionately, but the secondary reactance has been so far reduced, that the secondary current is now limited largely by resistance, and the secondary current and torque fall off quite uniformly as the speed increases, making this part of the torque-speed curve nearly straight. Curve I shows the characteristic behavior of a constant speed motor, since the speed drops only 3 per cent. at full load. The starting torque is a little less than half of that at full load.

Curve II is for the same motor with a considerable resistance added to the secondary circuit. It starts with a nearly maximum torque, but the speed between no load and full load drops 24 per cent. It, therefore, becomes to that extent a variable speed motor.

Curve III shows an intermediate condition between curves I and II. The motor starts with a good torque, and drops 9 per cent. in speed, between no load and full load.

The theoretical derivation of curves of this sort may be well illustrated graphically. For example, we will select a motor having a very low secondary resistance, and having a secondary reactance, at zero speed, six times as large. The secondary reactance is directly proportional to the frequency, and, therefore, to the slip.

In *Fig. 17*, P_2 represents the secondary resistance of this motor, and X_2 its reactance. Along the line of reactance are marked percentages of slip, from zero to 100 per cent. The horizontal distance from zero to any number represents, therefore, the secondary reactance for that slip. The secondary impedance for any slip is similarly represented by a line drawn from the top of the line P_2, to the number denoting the slip. The angle between the impedance line and

the resistance line is the angle by which the secondary current lags behind its generating E.M.F., or, as it is called, the angle θ.

Using the values of slip as abscissæ, and the corresponding impedance as ordinates, the curve of secondary impedance of *Fig. 17* was constructed. It may be noted that this

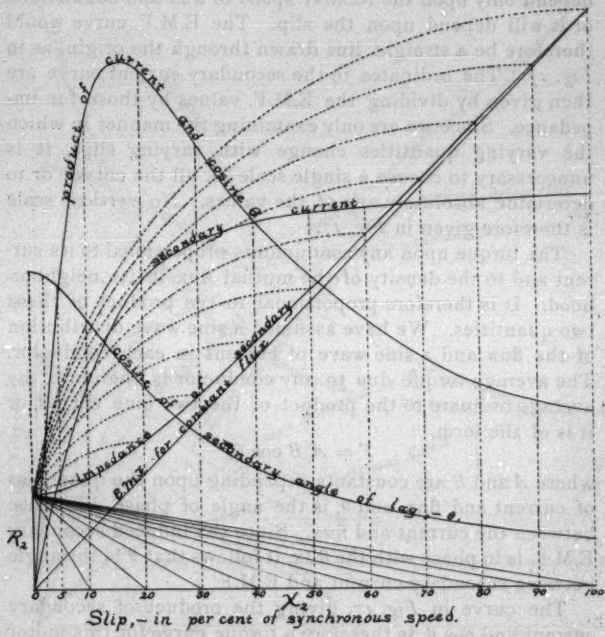


FIG. 17.

is very close to a straight line passing through the origin for a considerable distance. This indicates an impedance nearly proportional to the slip, over that range. We see also that between zero and 5 per cent. slip the impedance is practically constant. A curve of secondary power factors, or values of cos θ, is also drawn in *Fig. 17*.

If the secondary current can be determined in any manner, the torque characteristic may be immediately drawn. The secondary current depends only upon the E.M.F. and upon the impedance. The latter quantity we know. If we assume for the present that a constant quantity of flux enters the secondary, the E.M.F. induced will depend only upon the relative speed of flux and conductors, or it will depend upon the slip. The E.M.F. curve would therefore be a straight line drawn through the origin, as in *Fig. 17*. The ordinates to the secondary current curve are then given by dividing the E.M.F. values by those for impedance. Since we are only examining the manner in which the varying quantities change with varying slips, it is unnecessary to choose a single scale for all the curves, or to determine absolutely any of the values. No vertical scale is therefore given in *Fig. 17*.

The torque upon any conductor is proportional to its current and to the density of the mutual flux in its neighborhood. It is therefore proportional to the product of these two quantities. We have assumed a sine wave distribution of the flux and a sine wave of current in each conductor. The average torque due to any conductor is, therefore, the average ordinate to the product of the two sine curves, or it is of the form,

$$T = A\,B\cos\theta,$$

where A and B are constants depending upon the quantities of current and flux and θ, is the angle of phase difference between the current and flux. Since the induced secondary E.M.F. is in phase with the flux, it follows that θ is the angle between secondary current and E.M.F.

The curve in *Fig. 17*, giving the product of secondary current and cos θ, is therefore a torque curve for this motor under the assumed conditions. The only serious error made lies in taking the mutual flux threading both primary and secondary circuits as constant. In an actual motor this flux decreases with a rise in the secondary current. The part of the total primary flux that leaks past the secondary is practically zero when the secondary does not carry current. As the secondary current increases, the primary leakage in-

creases almost proportionally so long as the secondary power factor is near unity. A lag in the secondary current increases the leakage, so that as the secondary power factor decreases the primary leakage increases faster than does the secondary current.

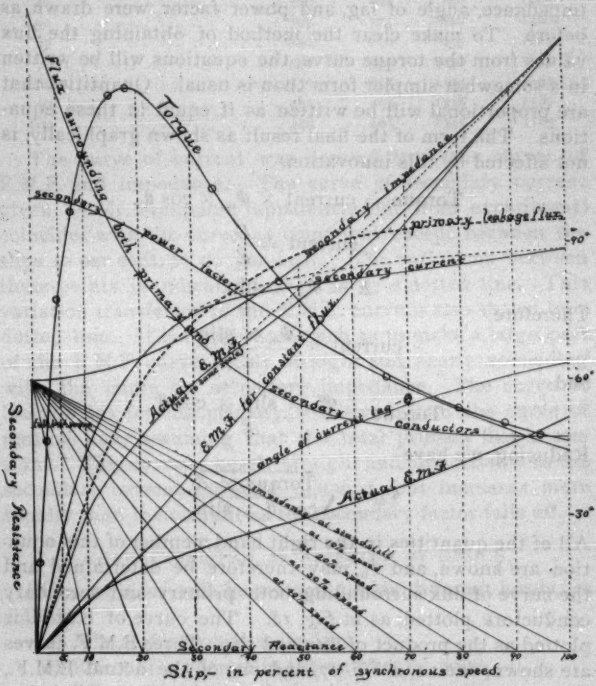


FIG. 18.

The exact relationship existing between these various items may be obtained by working backward the method used in connection with *Fig. 17*, starting with an experimental torque curve. In *Fig. 18* such a torque curve is given. The experimentally determined points on this curve

are determined by circles in the figure. This curve is also one given by Mr. Steinmetz (*Trans. A. I. E. E.*, Vol. 14, page 192). The secondary of the motor on which the observations were made had a reactance, at primary frequency, equal to three times its resistance. The curves of secondary impedance, angle of lag, and power factor, were drawn as before. To make clear the method of obtaining the flux values from the torque curve, the equations will be written in a somewhat simpler form than is usual. Quantities that are proportional will be written as if equal in these equations. The form of the final result as shown graphically, is not affected by this innovation.

$$\text{Torque} = \text{current} \times \Phi_m \times \cos\theta\,.$$

$$\text{current} = \frac{E}{Z}$$

$$E = \Phi_m \times \text{slip}.$$

Therefore,

$$\text{current} = \frac{\Phi_m \times \text{slip}}{Z},$$

and

$$\text{Torque} = \frac{\Phi_m^2 \times \text{slip} \times \cos\theta}{Z}$$

Reducing, we have

$$\Phi_m = \sqrt{\frac{\text{Torque} \times Z}{\cos\theta \times \text{slip}}}.$$

All of the quantities in the right hand member of this equation are known, and Φ_m may therefore be determined and the curve of flux surrounding both primary and secondary conductors plotted, as in *Fig. 18*. The curve of E.M.F. is plotted as the product of flux and slip. Three E.M.F. curves are shown, one as in *Fig. 17*, and one of the actual E.M.F., drawn for purposes of comparison with this. Another curve is drawn from the same data, but choosing such a scale as brought the E.M.F. at 100 per cent. slip to the same value as the impedance. This latter curve is marked "not to same scale."

A table showing the computed figures from which these curves were drawn is given on the next page.

TABLE FOR FIG. 18.

Slip	Cos. θ	Torque	Impedance	Φ
5	·995	795	603	3108
10	·980	1140	610	2670
20	·928	1240	648	2080
30	·857	1105	790	1739
40	·780	920	768	1505
50	·707	773	846	1360
60	·640	660	935	1269
70	·580	590	1030	1228
80	·530	535	1134	1194
90	·485	495	1238	1187
100	·446	465	1345	1185

The curve of current was derived from the curves of E.M.F. and impedance. The curve of secondary current given by Mr. Steinmetz (apparently not from experiment) coincides with the curve as computed, except between the slips 40 per cent. to 90 per cent. The variations between these points is indicated in *Fig. 18* by a dotted line. This variation transferred to the E.M.F. curve is also shown by a dotted line. It is in this case such as to make a large part of the E.M.F. curve nearly straight and nearly coinciding with the curve for secondary impedance. The curve of leakage flux for the primary is derived from the curve of mutual flux, assuming that the total primary flux is constant. This curve is nearly straight and proportional to the secondary current for some distance, but increases more rapidly than the current as the secondary factor falls off.

CHARACTERISTICS AS AFFECTED BY DESIGN.

This subject will be treated under different heads, as follows:

(*a*) Effect of a wound secondary.
(*b*) Effect of a change in the number of poles.
(*c*) Effect of a change in the air gap,
(*d*) Effect of a change in the number of slots.

(*a*) *Effect of a Wound Secondary.*

Reference has previously been made to only one form of secondary winding, *i.e.*, the so-called "squirrel cage" winding. This winding is of the most universal character. It

is adapted for operation with any number of poles, or for any number of phases. Currents may take any paths through it that the E.M.F.'s may indicate. In fact, the currents follow slight irregularities in the E.M.F. so closely that the current reactions damp out such irregularities almost entirely. It therefore follows that only a magnetic field of the simplest possible character, *i. e.*, approximating closely to a sine wave, may be sent through this secondary when there is relative motion between the secondary and the field. The squirrel cage secondary winding has, therefore, many exceedingly desirable qualities. It is, moreover, easy to construct, is mechanically strong, and very little likely to receive injury. It has but one unfortunate characteristic. It is all but impossible to make use of a variable resistance in its circuit.

We have seen that a motor with a slow resistance winding in its secondary, may have a high efficiency, and run at a constant speed for all loads within its rated output, but its starting effort is small. Conversely, a motor may be made with a higher secondary resistance, and its starting torque, therefore, may be almost anything that is desired, but the speed regulation is likely to be poor, and the efficiency not so satisfactory. It is quite necessary in many classes of service to make use of a motor that is efficient, that will run at a constant speed, and that will have a high starting torque. It is practically impossible to secure all of these advantages without using some form of variable secondary resistance. To accomplish this, the secondary is given a winding similar to that used for the primary, and the resistance is inserted in the lines, so as to close the circuit. This resistance may be a part of the motor, or it may be entirely separate from the machine, connection being made through slip rings.

The secondary winding must evidently be made such as is suitable for a particular number of poles. Given secondary conductors are always in series, and each must carry the same current as the others. The E.M.F. induced in a circuit is the sum of the E.M.F.'s induced in conductors lying at various points on the rotor. An irregularity in flux that is localized on the rotor does not at once generate a corres-

ponding localized current and the secondary reactions do not therefore smooth out the flux wave to any great extent. The magnetic reactions of the currents in a wound secondary are not therefore such as force a sine flux distribution. The starting current of a motor having a variable secondary resistance is much less for the same torque than that of a commercially practicable motor having a fixed secondary resistance.

The squirrel cage motors, as ordinarily put upon the market, have end rings made of special alloys, having considerable resistance, and their performance is a compromise which gives a fair efficiency and speed regulation, an exceedingly large starting current, and a good starting torque. There are no theoretical differences in the operation of the two classes of motors which involve a change in the treatment already given.

(*b*) *Effect of a Change in the Number of Poles.*

When a motor speed and frequency are both fixed, as is sometimes the case, the number of poles is arbitrarily determined. When either of these values is not specified, the number of poles may theoretically be any even number. Practically, it is found highly desirable to keep the number of poles as small as possible. If a change be made in the winding of a given motor frame, such as to increase the number of poles, it is obvious that the distance from pole to pole is reduced. But the magnetic leakage occurs along the air gap from one pole to the next, and a shortening of this path will, other things being equal, increase the leakage.

(*c*) *Effect of a Change in the Air Gap.*

A cardinal principle regarding the air gap is that it must be kept as short as possible, sufficient clearance being provided for mechanical safety. The air gaps of motors built at the present day are materially shorter than those of a few years since, owing to improved methods of manufacture. An increased air gap simply provides an increased area for leakage, and by demanding an increased magnetomotive force increases the actual leakage in two ways.

The effect upon the performance of a motor due to a wear of the bearings, and a consequent unequal air gap at the top and bottom of the machine, is of some interest. If the primary be wave wound, the E.M.F. of a single circuit is distributed around the primary ring, and may be quite unequal at different points. Where the reluctance of a magnetic circuit has been reduced by wear in the bearings, and a consequent decrease in the air gap, the flux will be greatly increased. Where the air gap has been increased, the flux is correspondingly lessened.

TABLE I.

RUNNING CONDITIONS OF TWO-PHASE VOLT MOTORS WITH SQUIRREL-CAGE ROTORS.

Horse-Power.	Speed in R. P. M.		Efficiency in Per Cent		Per Cent Power. Factor.		Max. Torque Terms of Full Load.
	Syn's	Load.	Half Load.	Full Load.	Half Load.	Full Load.	
1	1800	1690	77	80	72	88	2·6
3	1800	1690	75	80	72	88	2·7
5	1200	1130	79·5	82	76	89	2·6
7·5	1200	1130	80	82	81	91	2·75
10	1200	1130	80	82·5	81	91	3·2
15	1200	1130	83	84·5	82	90	2·8
20	1200	1130	84	84·5	80	91	2·7
30	900	845	85	85	81	90	2·8
40	900	850	86	86·5	78	90	
50	900	850	88	88	86	93	3·5

If a lap wound primary be used, each current circuit is localized, and since the primary flux is dependent upon the E.M.F. of a circuit, and that of all circuits is equal, the flux at a pole is independent of irregularities in the air gap.

There cannot therefore be unbalanced magnetic pulls on the rotor of a lap wound machine, while these might be serious in the case of one that is wave-wound.

(*d*) *Effect of a Change in the Number of Slots.*

When the number of poles is fixed, a change in the number of slots is a change in the quantity known as the "slots

per phase per pole," and if the total number of conductors remains the same, this also implies a change in the number of conductors per slot. These changes may alter the

TABLE II.

STARTING CONDITIONS OF TWO-PHASE CONSTANT SPEED MOTORS WITH SQUIRREL-CAGE ROTORS.

Horse-Power.	E. M. F.	Starting Torque.		Line Current.		Per Cent. Power Factor.
		Lbs. at One Foot Radius.	Terms of Full Load.	Amperes.	Terms of Full Load.	
1	200	7·2	2·4	14	5	60
3	200	23	2·5	40	5	62
5	200	51	2·2	55	4·2	60
5	130	23	1·	23	1·9	60
	100	12.5	·55	14	1·1	60
7·5	200	87	2·5	90	4·7	62
7·5	130	37	1.05	28	1·9	62
7·5	100	21	·60	23	1·2	62
10	200	131	2·8	140	5·6	62
10	130	54	1·15	59	2·2	62
10	100	32·5	·70	35	1·4	62
15	200	168	2·4	174	4·7	60
15	130	70	1·	74	2·6	60
15	100	42	·6	44	1·2	60
20	200	211	2·3	230	4·6	60
20	130	89	·97	97	1·9	60
20	100	50·5	·55	58	1·2	60
30	200	495	2·3	345	4·6	63
30	130	176	·95	145	2	63
30	100	96	·52	86	1·2	63
40	200	584	2·4	525	5·25	65
40	130	248	1·0	220	2·0	65
40	100	136	·55	132	1·2	65
50	200	792	2·6	672	6·0	65
50	130	335	1·1	280	2·20	65
50	100	182	·6	170	1·35	65

behavior of the motor in two ways. First, an increase in the number of slots per phase per pole means that less M.M.F. is added to a circuit by the currents lying in one

slot, and that the change in magnetic distribution from zero to a maximum is less abrupt, and the wave of magnetism belonging to a single pole is made to approximate more closely to a sine curve. This is highly desirable. Second, an increase in the number of slots means of winding that is more distributed, while a decrease in the number of slots involves a condensation of the windings. The former decreases the self-induction, and the latter increases it. For the second reason therefore it is also desirable to use a large number of slots.

The expense of shop construction is considerably increased by an increased number of slots, so that an actual motor is in this respect a compromise between initial cost and advantages of operation.

Experimental Constants from Typical Motors.—To illustrate the actual behavior of commercial machines, there will be given some test records taken from typical motors. These figures are for the most part from machines that were constructed during the year 1900. The first table shows the running conditions of a line of two phase, 200 volt motors having squirrel cage secondaries. They are designed to have a considerable starting torque, and the short circuiting end rings are made of an alloy of copper and german silver. The speed regulation and efficiency are as good as can be obtained under the circumstances.

The starting torque, starting current, and power factors of the same motors, and the variations in these with changes in the impressed pressure, are shown in the second table. It is seen that a very high starting torque is obtained if the full pressure is immediately thrown on, but this is only done by drawing from five to six times the normal primary current. The starting torque given by half the normal pressure, and something more than normal full-load current, is a little over half of the full-load torque. In the practical operation of these motors it is intended that the starting shall be accomplished at a reduced pressure.

The amount of current that a motor consumes when running without load, and the power factor at which this current is drawn, are quite important factors in determining

its value. An average giving test results of this sort has been made for each of several classes and sizes of motors.

Taking the test records of twenty-eight different motors, of a type similar to those for which data are given in Tables 1 and 2, the motors being 60-cycle machines, and having capacities ranging from ½ to 200 horse-power, an average shows that 37 per cent. of the normal full-load current is drawn from the line, with the motors running unloaded. At the same time 5·5 per cent. of the rated power output is the average power consumed. This corresponds to a no-load power factor of about 13 per cent.

Similar figures taken from a line of two-phase 25-cycle motors gave for eleven machines, ranging in size from 1 to 20 horse-power, 34 per cent. of the full-load current, and power amounting to 5·3 per cent. of the rated output as required for light running. From thirteen similar machines of larger size, ranging from 30 to 500 horse-power, the figures were 33 per cent. of the normal full-load current and 4·1 per cent. of the power of the rated output of the motors.

Of three-phase motors averages were taken of the results of tests on twenty machines of capacities from ¼ to 100 horse-power. The average drop in speed of these motors from synchronous speed to that at full load was 5·5 per cent. The quarter horse-power machine required 85 per cent. of its normal full-load current when running light, and under these conditions used 61·5 watts, or 33 per cent. of its power output. Eight of the machines of capacities, from 2 to 15 horse-power, required 28 per cent. of the full-load current and 5·56 per cent. of the power output, and eleven of the larger sized motors, of capacities from 20 to 100 horse-power, required 26 per cent. of the full-load current and 5 per cent. of the power output for no-load running.

Similar figures are given in detail below for ten high-resistance motors designed for such duty as is required by cranes, hoists, etc.

The motors from which the figures given in the following table were taken, are all three-phase machines. A wide range of performance is found between individual motors of the same general character.

Horse-Power.	Cycles.	Speed Drop per cent. Synchronous to Full Load.	Per Cent. of Full Load Current to Run Light.	Per Cent. Power Factor to Run Light.
		Per Cent.	Per Cent.	Per Cent.
5	25	5	54'5	11'6
10	25	2'7	54	10'4
20	25	6	66	12'3
40	25	7'5	36	6
3	30	22'6	57'5	11'3
5	30	4	53'4	12'7
10	30	4'3	47'5	9'2
5	60	9	55	10'5
10	60	17'8	34'5	8'9
20	60	6'5	65	8

The results of tests that have been given in the preceding pages will serve to indicate about the sort of performance to be expected from induction motors at the present day. The usual limits that are set by a good designer for the variable factors which he controls, will be given in the next section.

Design Constants from Typical Motors.—The design constants that are given in this section were computed from machines built in this country during the year 1900. The writer was particularly fortunate in being able to secure this data, which is of a nature not usually available. Partial dimensions were obtained for nineteen different machines. Not all of the desired dimensions could be had in every case, and some of the constants could not be computed for all the motors. A considerable amount of material was, however, obtained. The motors are numbered consecutively, and a given number indicates the same motor on each of the pages.

The first seven motors have wound secondaries, and the remaining twelve motors have short-circuited or squirrel-cage secondaries. The motors of each group are arranged in order, according to size. The size is rated upon the diameter of the primary bore, and not upon the power output. All the machines except Nos. 8 and 16 are three phase. Nos. 8 and 16 are wound for two phases.

Motors Nos. 10, 14, and 15 are what is known as variable pole motors. The primary winding of one of these machines is arranged so that while at full speed it is *Y* connected with two circuits per phase, and a given number of poles; it may be changed by proper switching devices to a delta-connected

Motor Number.	Outside Diameter Primary.	Inside Diameter Primary.	Outside Diameter Secondary.	Inside Diameter Secondary.	Horse-Power.	Speed in R.P.M.	Number of Poles.	Cycles per Second.	Primary E.M.F.	Length of Laminations Parallel to Shaft.	Length Occupied by Ducts.
1	26	19·07	19	14	16	720	10	60	440		
2	26	19·07	19	14	20	600	12	60	440	10	·75
3	32	24·08	24	18	30	450	16	60	440	9	·75
4	32	24·09	24	18	30	450	16	60	440	9	·75
5	32	24·08	24	18	50	450	16	60	440	14	·15
6	40	32·0 9	32	26	75	450	16	60	440	10·5	1
7	40	32·09	32	26	150	600	8	40	550	16	2
8	17	12·06	12	8	8	1333	6	66	550	6·75	0
9	21	15·07	15		7	480	10	40	110	7·5	0
10	40	32·09	32	26	50	480	10	40	550	7·5	·75
11	40	32·09	32	26	75	250	12	25	220	10·5	1
12	42	32·09	32	24	75	500	6	25	440	10·5	1
13	42	32·09	32	24	75	500	6	25	440	10·5	1
14	50	40·1	40	35	100	400	12	40	550	10	·75
15	50	48·⅛	48	40	150	300	16	40	550	17	1·5
16	58	48·⅛	48	40	560	450	16	60	190	17	1·5
17	70	60·	59·86		700	400	18	60	2200	16	1·5
18	76	60·	59·86	46	1200	375	8	25	352	24	3
19	84	70·25	70	62	250	187	16	25	3000	12	1·5

motor, having one circuit per phase, and twice the number of poles as before. Synchronous speed with the latter connection is then half of synchronous speed with the former connection. The E.M.F. produced per circuit is 318 volts at full speed with the *Y* connection, and 550 volts at half speed with the delta connection. Since the pressure in the

former case is generated with half the number of conductors in series, and with the flux moving at twice the speed, as in the latter case, the air gap induction densities must also be in the ratio of 318 to 550. The various flux densities given in the tables for these machines are for full speed. These

Motor Number.	Primary.										Connection of Secondary.
	Number of Slots.	Slots per Phase per Pole	Width of Slot.	Depth of Slot.	Thickness of Insulation.	Connection.	Circuits per Phase.	Number of Conductors.	Conductors in Series.	Size of Conductors.	Mean Width of Tooth.	
1	120	4	·28	1·25	·055	Y	1	1200	400	Twin No. 12	·236	Wound.
2	96	—	·28	1·25	·055	—	—	—	—	—	·386	
3	144	3	—	—	—	—	—	—	—	—	—	
4	144	3	·27	1·30	—	—	—	—	—	—	·284	
5	144	3	·27	1·30	—	—	—	—	—	—	·284	
6	144	3	·34	1·30	·05	△	1	864	288	Twin No. 9	·386	
7	144	6	·375	1·25	·068	△	1	576	192	·1 x 4 bar.	·355	
8	72	4	·28	1·25	·05	2φ	2	3456	864	No. 18 B. & S.	·304	Squirrel Cage.
9	90	3	·28	1·25	·049	△	2	1620	270	No. 12 B. & S.	·292	
10	120	4	·38	1·50	·05	Y	2	1920	320	No. 8 B. & S.	·500	
11	144	4	·36	1·60	·062	△	1	576	192	·1 x ·6 bar	·375	
12	108	6	·40	1·60	·06	△	1	864	288	4 Twin No. 16 B.W.G.	·580	
13	108	6	·40	1·60	·068	Y	1	648	216	·6 x ·075 bar.	·580	
14	144	4	·44	1·70	·05	Y	2	1728	288	3—No. 10 B. & S.	·470	
15	192	4	·42	1·70	·05	Y	2	1536	256	4 Twin No. 13	·397	
16	192	6	·40	1·75	·05	2φ	4	384	48	2—·7 x 15	·417	
17	216	4	·49	1·70	·10	Y	1	864	288	⅛ x ·6 bar	·410	
18	144	6	·60	1·70	·05	△	2	288	48	2—·65 x 2	·845	
19	192	4	·56	2·20	·10	Y	1	2688	896	3—No. 12 B.W.G.	·620	

should be multiplied by 1.73 ($\sqrt{3}$), to give the corresponding densities at half speed. It should be noted that the densities as set down in the tables are quite low as compared with those for the other motors, except for Nos. 8 and 9, which are of quite small capacity.

In studying the motor constants here given, and in making comparisons between them, it should be noted that a constant for any individual machine is subject to what may be termed accidental variations. Even with the motors of large size, these constants vary greatly in some cases. Such

Motor Number.	Secondary.												Peripheral Speed in Feet per Minute.
	Number of Slots.	Slots per Phase per Pole.	Width of Slot.	Depth of Slot.	Thickness of Insulation.	Connection.	Circuits per Phase.	Conductors per Slot.	Number of Conductors.	Conductors in Series.	Size of Conductor.	Mean Width of Tooth.	
1	—	—	·36	1·13	·08	Y	2	2	—	—	2—·1 × ·4	—	3390
2	72	2	·36	1·13	·08	Y	2	2	144	24	2—·1 × ·4	·42	2820
3	96	2	·36	1·13	·08	Y	2	2	192	32	2—·1 × ·4	·39	2830
4	96	2	·36	1·13	·06	Y	1	4	384	128	·1 × ·4	·39	2830
5	96	2	·36	1·13	·18	Y	2	2	192	32	2—·1 × ·4	·39	2830
6	192	4	·285	1·03	·055	Y	1	2	284	128	·175 × ·35	·22	3770
7	108	—	·32	1·33	—	—	—	—	—	—	—	·574	5530
8	—	—	·55	·63	—	P	Squirrel Cage.		—	2	·5 × ·5	—	4190
9	47	1·57	·55	·48	—	P			47	2	·5 × ·35	·425	1881
10	99	3·3	·562	·47	—	P			99	2	—	·438	4020
11	99	2·75	·55	·58	—	P			99	2	—	·448	2·90
12	99	5·5	·552	·47	—	P			99	2	—	·438	4200
13	99	5·5	·562	·47	—	P			99	2	—	·438	4200
14	133	3·7	·562	·47	—	P			133	2	—	·371	4190
15	186	3·88	·55	·63	—	P			186	2	·5 × ·5	·25	3760
16	186	5·8	·562	·94	—	P			186	2	2—·35 × ·5	·233	5660
17	210	3·9	·562	·4	—	P			210	2	—	·32	6280
18	210	8·77	·55	·50	—	P			210	2	—	·334	5880
19	255	5·3	·56	·47	—	P			255	2	—	·30	3430

variations are mainly due to the necessity of making a given design conform to the standards of the shop in which it is to be constructed. Thus, while a particular size of tooth and slot may be perfectly suited to one motor, and not so well adapted to one having a different capacity or speed,

commercial conditions may demand that both motors be built with the same tools and from similar stampings. The design constants in such a line of machines cannot be kept entirely uniform, and some of the motors turned out by the shop are likely to be considerably better than others of the same general type.

Motor Number.	Flux per Pole at No Load in Maxwells.	Flux Density per Square Inch.			
		Average Over Air Gap at No Load.	Primary.		
			Average in Core — Over Section Carrying Flux.	Average in Teeth.	Maximum in Teeth — Assuming a Sine Wave.
1	495,000	—	—	—	—
2	—	—	—	—	—
3	—	—	—	—	—
4	—	—	—	—	—
5	—	—	—	—	—
6	1,209,000	20,100	26,500	40,500	63,700
7	3,360,000	19,100	49,400	41,700	65,500
8	446,000	11,000	30,400	21,100	33,200
9	477,000	13,460	23,500	26,900	42,300
10	1,168,000	11,900	27,100	22,200	34,900
11	2,142,000	27,100	33,800	56,100	88,400
12	2,890,000	18,170	50,500	32,400	51,000
13	2,764,000	17,380	45,000	30,900	48,700
14	1,298,000	13,400	24,800	27,600	43,400
15	1,460,000	10,000	16,100	21,300	34,400
16	3,090,000	21,200	34,700	44,500	70,000
17	3,425,000	22,300	39,700	53,500	84,100
18	13,790,000	28,000	57,100	48,000	75,500
19	3,620,000	25,000	41,000	51,600	81,400

The item in which there is the best agreement between the various machines is that of circular mils per ampere. This was figured on the assumption that the product of the full-load efficiency and the full-load power factor would be ·8 in all cases. This brings the values given in the tables a

little too high for the small machines and a little too low for those of larger size, but the values are approximately correct as an average.

Considerable variation is found in the volts generated per foot of active conductor. This quantity depends upon

Motor Number.	Flux Density in Secondary.			Volts per Foot of Active Primary Conductor.	Approximate Circular Mils per Ampere in Primary Conductor.	Per Cent. of Primary Circumference Occupied by Mean Primary Teeth.	Per Cent. of Secondary Circumference Occupied by Mean Secondary Teeth.	Ratio of Section of Primary Core to Section of Secondary Core.
	Average in Core—over Section Carrying Greatest Flux.	Average in Teeth.	Maximum in Teeth—Assuming a Sine Wave.					
1	—	—	—	—	670	46	—	1·61
2	—	—	—	—	—	58	54	1·61
3	—	—	—	—	—	—	52	—
4	—	—	—	—	—	51	52	1·41
5	—	—	—	—	—	51	52	1·41
6	35,700	53,400	84,000	1·94	500	53	44	1·34
7	80,000	34,300	54,000	2·45	600	49	64	1·63
8	—	—	—	—	600	52	—	·95
9	—	35,600	56,000	·63	660	51	44	—
10	26,300	30,700	48,300	1·22	700	57	44	·99
11	52,900	68,600	107,600	1·45	720	51	45	·94
12	45,700	46,800	73,700	1·94	640	59	44	·90
13	59,500	44,800	70,600	1·86	780	59	44	1·32
14	38,500	37,800	59,400	1·44	640	52	40	1·60
15	15,400	36,000	56,600	·96	570	49	31	·96
16	36,200	81,000	127,000	3·07	780	51	29	1·04
17	—	70,000	110,000	3·63	560	46	36	—
18	60,500	83,400	131,000	4·18	640	63	38	1·05
19	54,300	80,000	125,000	2·20	790	52	35	1·32

the peripheral speed of the flux, and upon the air-gap density. The motor (No. 18) having the largest number of volts per foot of active conductor, and also having a very high induction density in all parts of the magnetic circuit, is not one for which the design here given can be called successful.

Motor No. 11 was the one chosen for the detailed study of Part One of this thesis. That motor had the highest induction density in the primary teeth of any here given; it was therefore particularly suitable for such a study. The results given in the table are, in the case of this motor, practically those found by the more elaborate study of Part One. If the results of the simple computations used in the preparation of these tables were accurate for such a motor as No. 11, they are even better for motors in which the iron is not worked to such high densities.

The induction densities found in these machines is in all cases very moderate as compared with those found in other classes of machinery. This is necessarily so, since every part of the magnetic circuit carries an alternating flux, and since, as we have seen, the motor is peculiarly sensitive to magnetic leakage.

[*To be concluded.*]

RADIO-ACTIVE CHANGE.

Messrs. Rutherford and Soddy, in a recent paper published in the *Philosophical Magazine*, discuss the products of radio-active change and their specific material nature, the synchronism between the change and the radiation, the material nature of the radiations, the law of radio-active change to chemical change, and the energy of radio-active change and the internal energy of the chemical action. Radio-activity, according to our present knowledge, they say, must be regarded as the result of a process which lies wholly outside the sphere of known controllable forces and cannot be created, altered or destroyed. Like gravitation, it is proportional only to the quantity of matter involved. Assuming that an atom of radio-active matter cannot produce less than one and not more than 200 "rays" or projected particles, the authors deduce 10^8 gram cal. per gram as the least possible estimate of the energy of radio-active change in radium. As the reaction which sets free more energy for a given weight than any other chemical change known, namely, the union of hydrogen and oxygen, liberates only approximately 4,000 gram cal. per gram of water produced, the energy of radio-active change must be at least 20,000 times, and may be 1,000,000 times as great as the energy of any molecular change. Calculating the rate at which this store of energy is radiated, the authors deduce that in 1 gram of uranium and thorium less than a milligram would change in a million years. In the case of radium, however, this amount must be changing per gram per year. Therefore, the "life" of the radium cannot be more than a few thousand years, on this minimum estimate, and it appears certain that the radium present in a mineral has not been in existence as long as the mineral itself, but is being continuously produced by radio-active change.—*Electrical World.*

Graphics of Carbon Disulphide, With Formulas and Vapor Table.

By Thomas M. Gardner.

To consider the properties of this vapor from the graphical standpoint and thus be able to see more clearly its value as a working fluid for heat engines, it was necessary to calculate the entropy values for a series of different temperatures and pressures, as there was no suitable table at hand. These several quantities in tabular form, with their equations, are given at the end of this article. Plate I consists of curves of the properties so determined.

The second plate is a temperature-entropy diagram with the constant-volume lines of McFarlane Gray, which show the entropy in a given volume at any temperature and pressure and the heat units contained. If a given volume of vapor of given state and pressure with corresponding temperature be taken, the total entropy will be the specific entropy of the liquid, to which is added the specific entropy of the vapor times its quality, both being multiplied by the weight of the working fluid. If now the pressure and temperature is diminished, v being constant, the entropies and the weight of the vapor are changed so that the retained energy is less than the original amount; we have

$$dE = f(t, \Phi, m).$$

This process occurs during the exhaust of the steam-engine.

Two different methods of constructing this chart may be followed: one a graphical method as devised by Capt. Sankey, and the other by comparison of the constant volume with the specific volume corresponding to that temperature minus the volume of 1 pound of its liquid. Since the increase in the volume is proportional to the latent heat r, and

$$r / T = \Phi v,$$

Φv is proportional to the increase in volume, the temperature during the operation being constant, we thus can tell

the amount of heat weight present in the retaining vessel under the stated conditions, and so are able to lay out a curve.

Hyperbolic Expansion.—The constant-area curves of Plate III are drawn in the following manner: Different volumes of 1 pound of working fluid, at a common pressure, but with varying initial qualities, are assumed. Then, according to the formula,

$$pv = c,$$

the different expansion curves were drawn with the saturation curve as a reference line. It is seen that the amount of moisture during such an increase of volume decreases.

$$pv = p'v', \quad \frac{pv}{p'} = \frac{pbs}{p'}$$

Now

$$x = \frac{v'}{s'} = \frac{pbs}{p's'}$$

$$\varphi = \varphi_1 + x\frac{r}{T}$$

Substituting, we have

$$\varphi = \varphi_1 + \frac{pbs}{p's'}\varphi_v$$

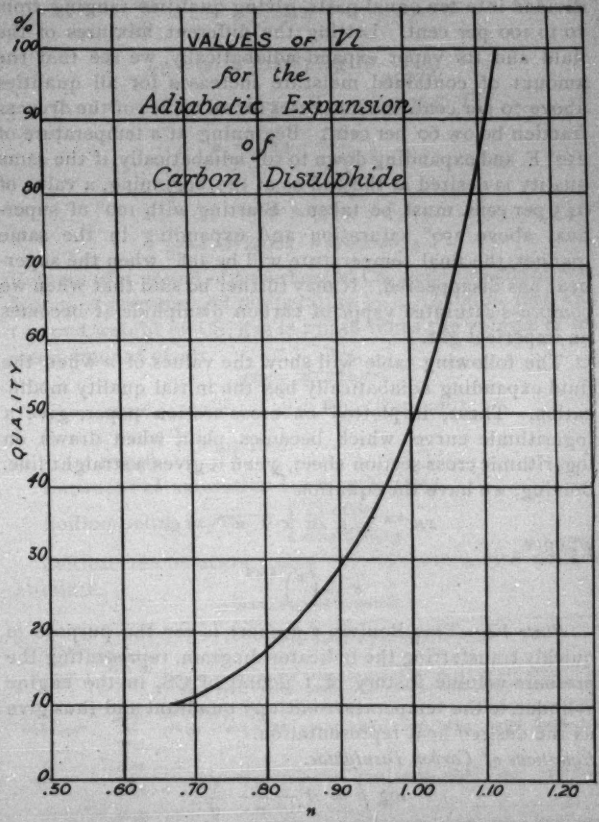
$$\varphi = \varphi_1 + \frac{c\,\varphi_v}{p's'}.$$

The total entropy of the volume considered is equal to the entropy of the liquid plus a quantity which is constant multiplied by the entropy of the vapor divided by the product of the absolute pressure times the specific volume; since for ordinary pressures the volume of the liquid may be neglected.

When the expansion of superheated vapor is considered according to $pv = c$, T being constant, all heat added is represented by external work, and the intrinsic energy is constant, so the entropy will be constant.

Constant Quality Curves.—Since the distance between the two entropy curves is proportional to r / T and represents the heat energy due to the increase in volume while the pound of liquid is evaporating till it is dry and saturated at that pressure, the quality is easily transferred from the

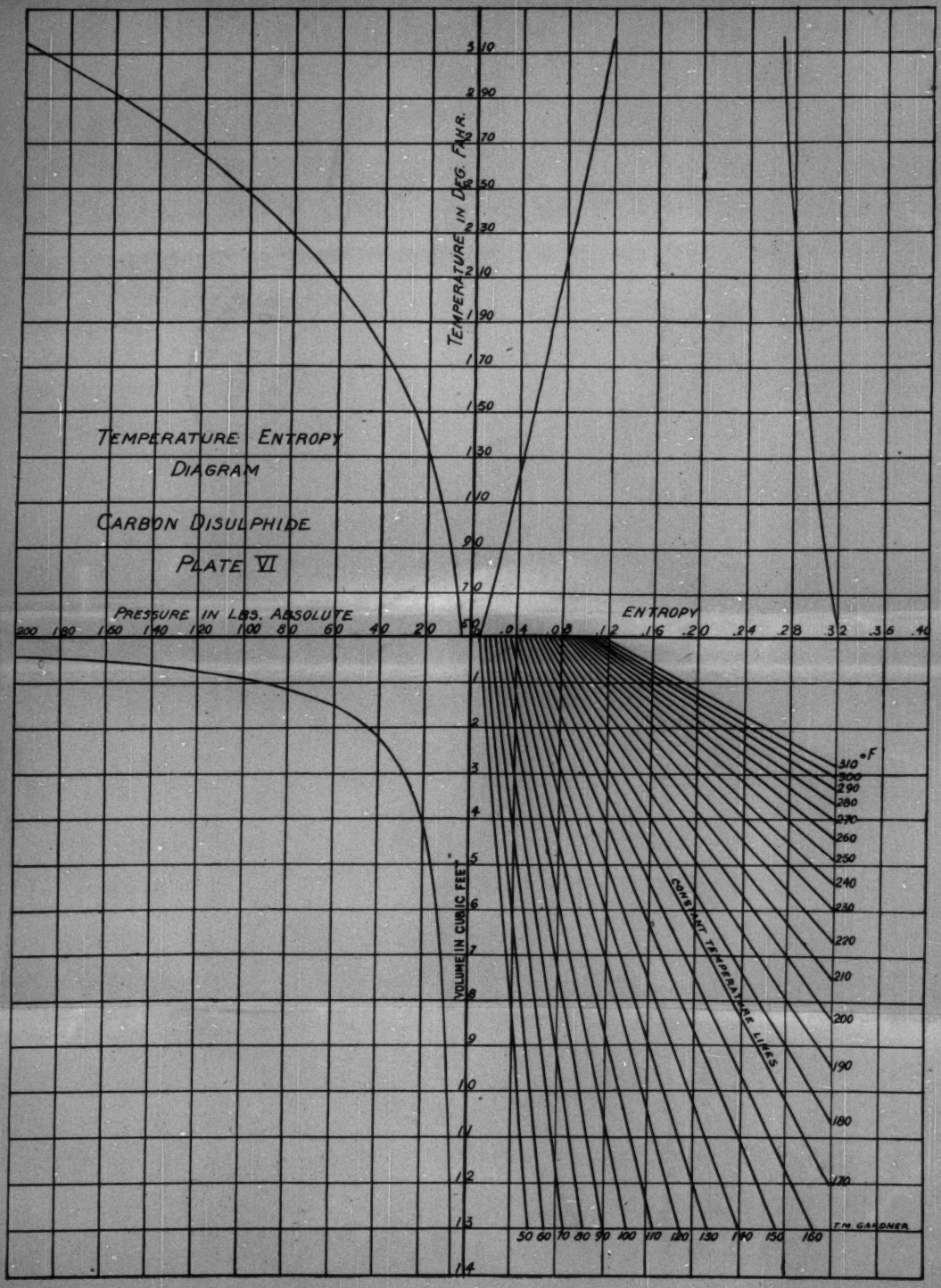
temperature entropy to the pressure-volume diagram. We have a series of curves with exponents the same. The expression is $pv^{1.0568} = c$.

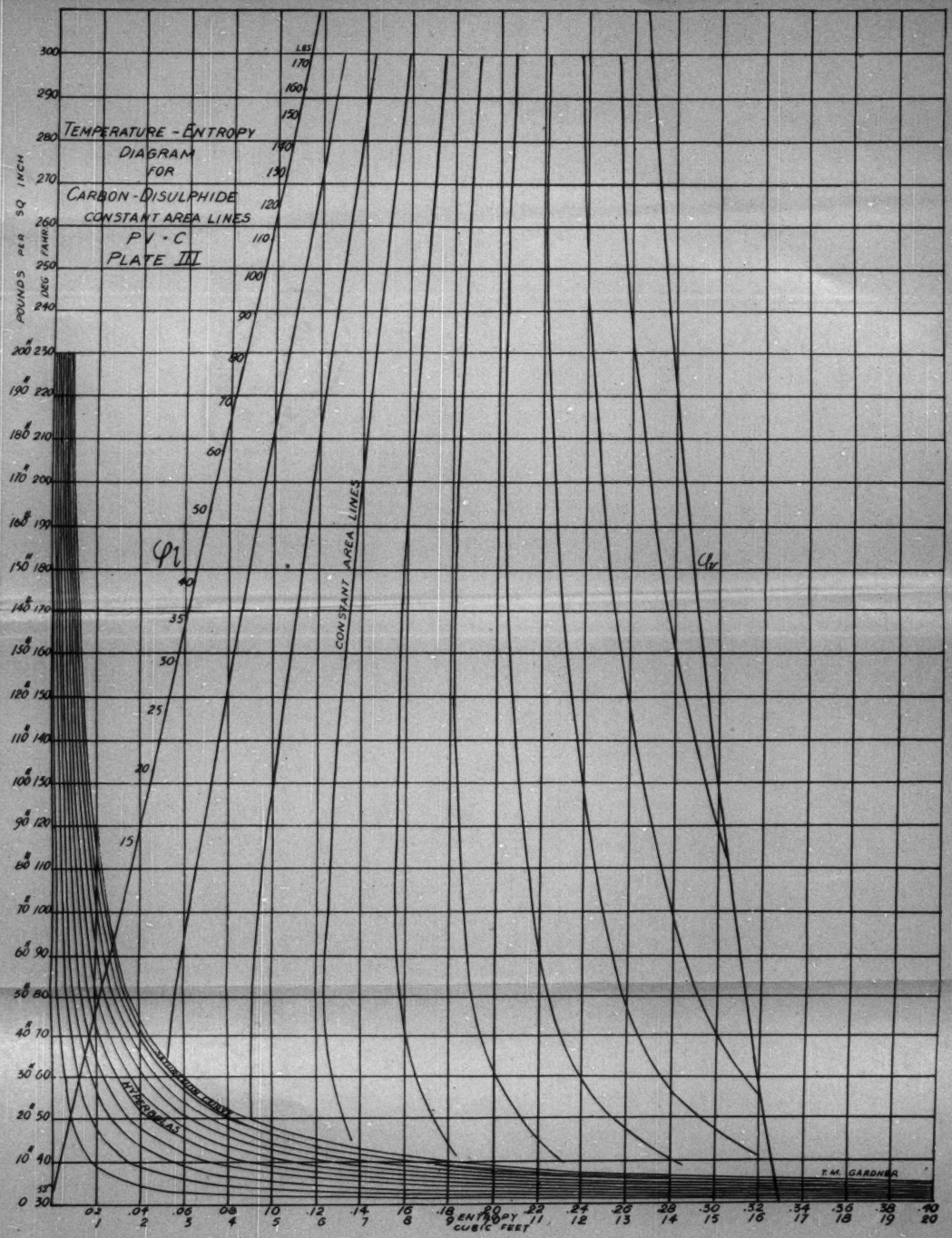


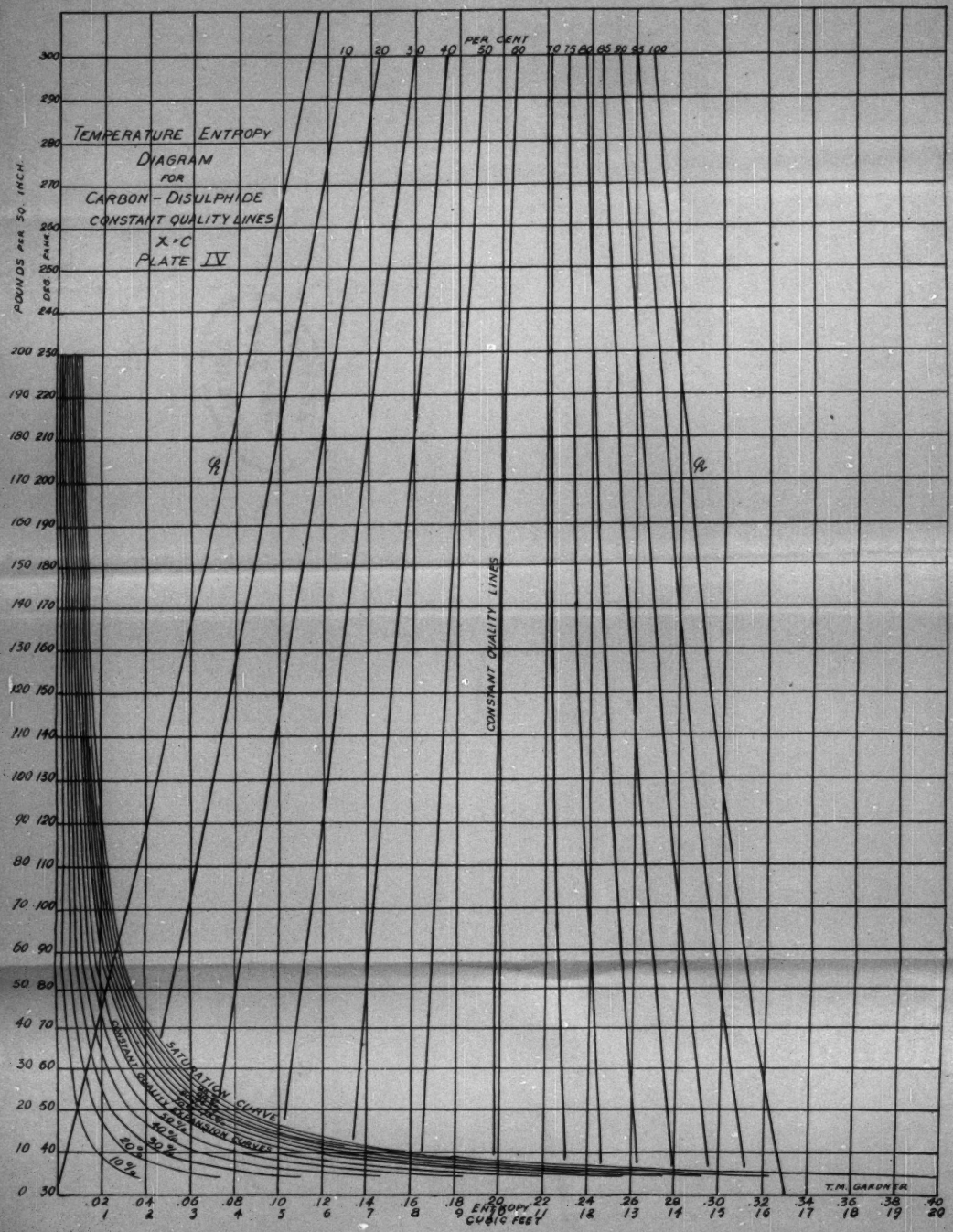
Adiabatic Expansion.—In this theta-phi diagram constant entropy lines are drawn, from which we may obtain the quality of the vapor, by taking the distance from the entropy

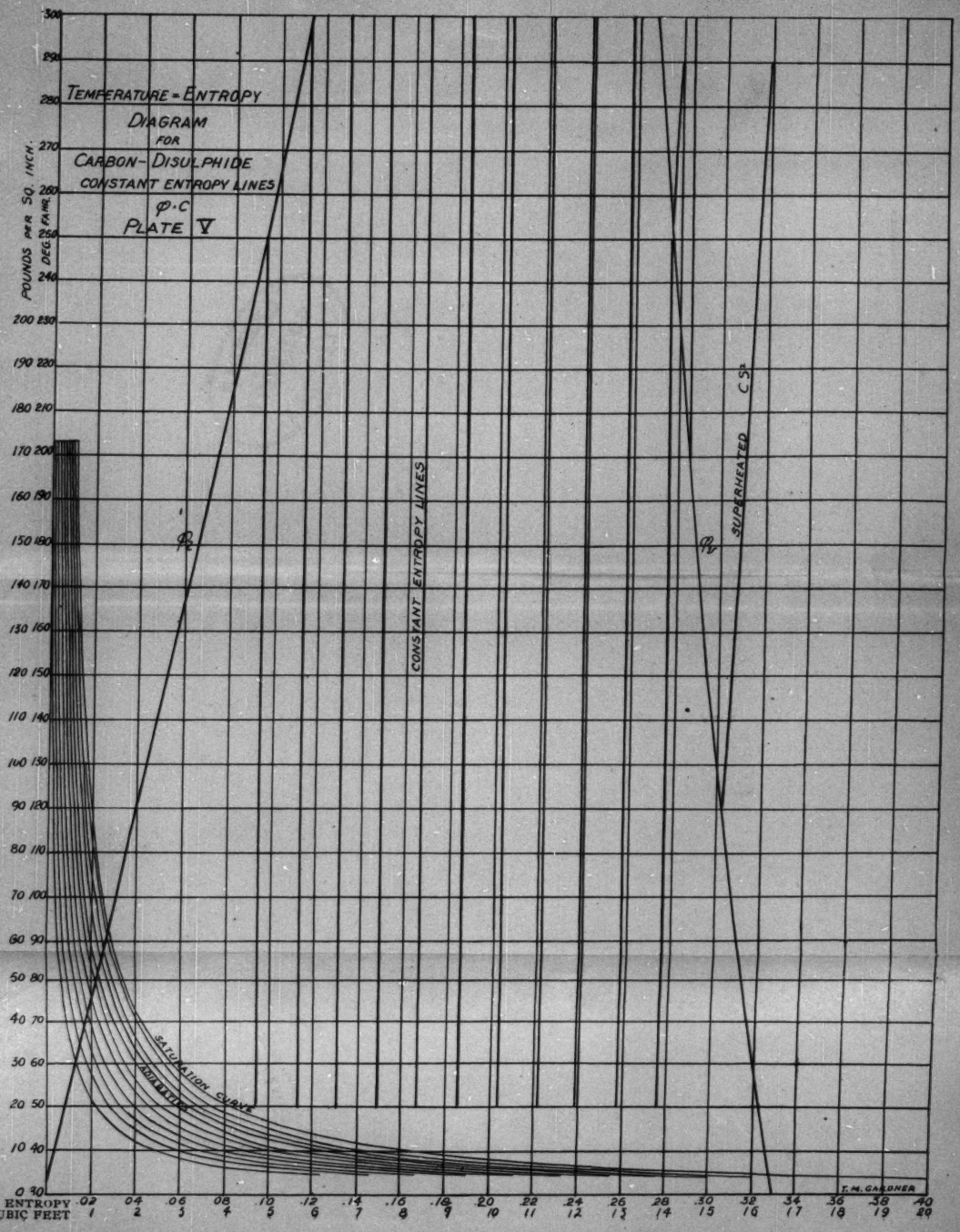
PROPERTIES OF SATURATED VAPOR OF CARBON DISULPHIDE.

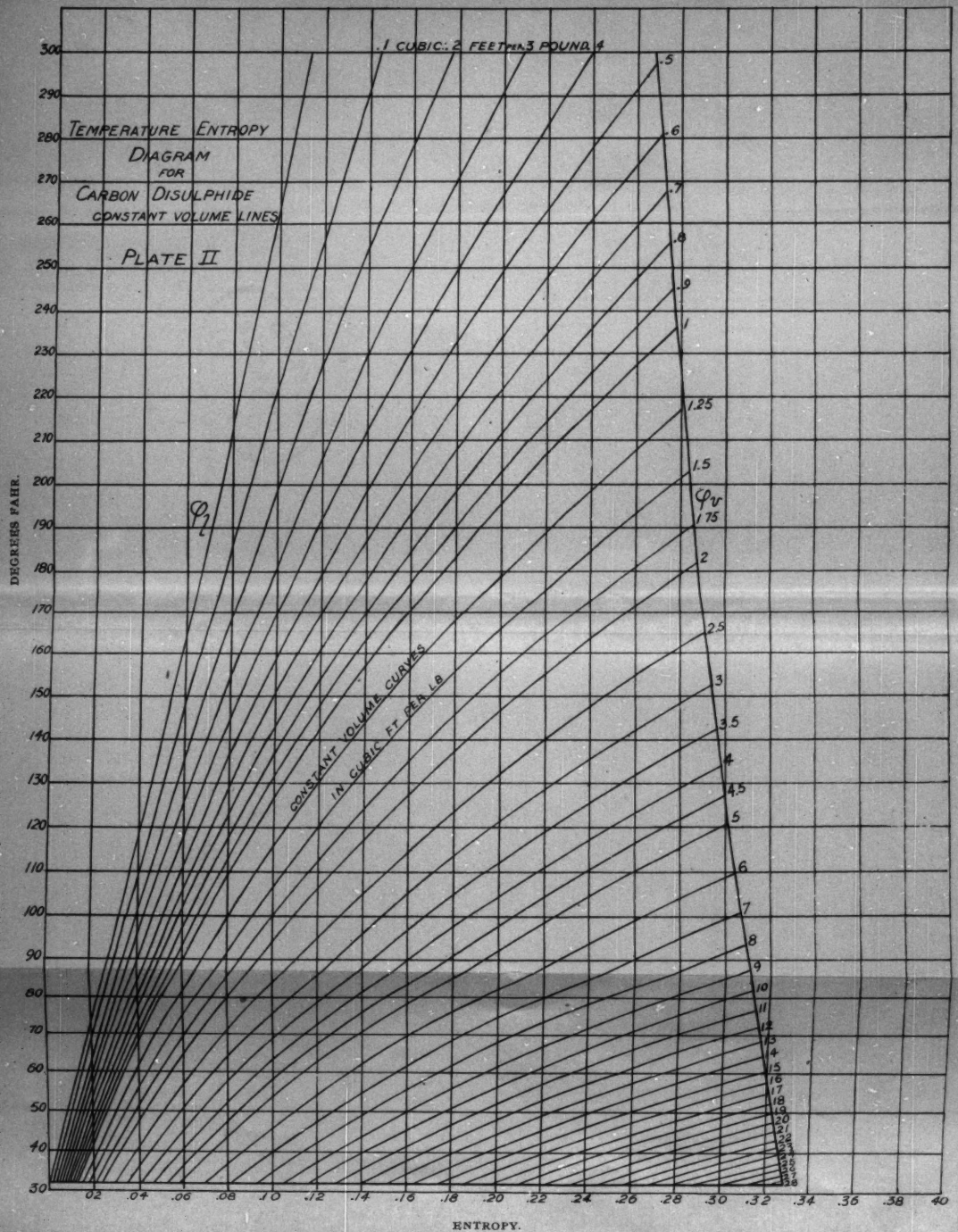
Deg. Fah.	Absolute Scale.	Lbs. Per Sq. In.	Lbs. Sq. Ft.	Heat Liquid. q	Total Heat. A	Heat of Vaporization. τ	Internal Latent Heat. ρ	External Latent Heat. apu	Entropy of Liquid. $\phi_1 = \int_c \frac{dt}{T}$	Difference per Deg.	Entropy of Vapor. $\phi_v = \frac{r}{T}$	Dif. per Deg.	Total Entropy. ϕ	Dif. per Deg.	$\frac{dp}{dt}$	$\tan \frac{dp}{dt}$ from a	Specific Vol. v	Vol. of Liquid. Cu. Ft. v_1	Wt. of 1 Cu. Ft. Vapor. $\frac{1}{v_1}$	Deg. Fah.
32	492·66	2·46870	355·492	·000018	161·9976	161·9976	149·0573	12·9403	·000000	. . . .	·388822	. . . .	·388822	. . . .	·062733	3° 35′	28·33259	·012570	·035295	32
35	495·66	2·66280	393·448	·706078	152·4336	161·7276	148·7110	13·0166	·001430	·001477	·386287	·000865	·387717	·000368	·066765	3° 49′	26·42867	·012596	·037846	35
40	500·66	3·01431	431·663	1·8847	163·7513	161·8666	148·1927	13·0739	·003801	·003874	·384208	·000835	·385999	·000361	·073924	4° 14′	23·44588	·012640	·042514	40
45	505·66	3·40490	490·311	3·0656	163·8576	160·7919	147·5402	13·2518	·006159	·006171	·381984	·000824	·381443	·000353	·082704	4° 40′	21·03797	·012684	·047529	45
50	510·66	3·83180	551·780	4·2489	164·5524	160·3035	146·9643	13·3392	·008500	·008568	·313914	·000816	·322414	·000345	·089667	5° 8′	18·82083	·012727	·053133	50
55	515·66	4·30370	619·733	5·4343	165·2357	159·8313	146·3165	13·4849	·010829	·010965	·309837	·000803	·320736	·000337	·096905	5° 31′	16·94157	·012771	·059127	55
60	520·66	4·82190	694·357	6·6121	165·9076	159·2854	145·6309	13·6545	·013144	·013314	·305930	·000793	·319074	·000330	·108038	6° 10′	15·31182	·012815	·065299	60
65	525·66	5·38970	776·121	7·8121	166·5679	158·7558	145·0475	13·7083	·015446	·015660	·302012	·000783	·317458	·000323	·118745	6° 46′	13·87437	·012859	·072703	65
70	530·66	6·01050	865·516	9·0344	167·2169	158·2125	144·3953	13·8172	·017735	·017958	·298142	·000774	·315877	·000316	·129696	7° 23′	12·43299	·012902	·080431	70
75	535·66	6·68860	963·267	10·1991	167·8545	157·6534	143·7321	13·9233	·020012	·020255	·294330	·000765	·314330	·000309	·141375	8° 3′	11·03478	·012946	·089009	75
80	540·66	7·42550	1069·270	11·3959	168·4895	157·0846	143·0342	14·0604	·022276	·022452	·290542	·000756	·312818	·000302	·153446	8° 44′	10·03682	·012990	·097625	80
85	545·66	8·22900	1184·976	12·5951	169·0951	156·5000	142·3707	14·1293	·024528	·024650	·286809	·000747	·311337	·000 96	·167039	9° 29′	9·21871	·013013	·107645	85
90	550·66	9·09680	1309·965	13·7961	169·6382	155·9217	141·6729	14·2488	·026769	·026849	·283118	·000738	·309887	·000290	·181002	10° 16′	8·46584	·013077	·118151	90
95	555·66	10·0385	1445·541	15·0002	170·2900	155·2898	140·9638	14·3260	·028998	·029078	·279469	·000729	·308467	·000284	·195828	11° 6′	7·74196	·013121	·129178	95
100	560·66	11·0566	1591·147	16·2061	170·8701	154·6640	140·2439	14·4211	·031214	·031294	·275865	·000725	·307080	·000277	·211505	11° 57′	7·05999	·013164	·141643	100
105	565·66	12·1552	1750·340	17·4145	171·4392	154·0246	139·5112	14·5132	·033421	·000441	·272292	·000714	·305713	·000272	·228052	12° 57′	6·46409	·013208	·154701	105
110	570·66	13·3316	1920·758	18·6250	171·9964	153·3714	138·7684	14·6030	·035619	·000439	·268762	·000706	·304381	·000266	·245491	13° 48′	5·92817	·013252	·168686	110
115	575·66	14·5744	2098·717	19·8379	172·5423	152·7044	138·0142	14·6902	·037802	·000437	·265268	·000698	·303070	·000262	·263177	14° 41′	5·44901	·013296	·183184	115
120	580·66	15·9786	2300·922	21·0529	173·0766	152·0233	137·2487	14·7750	·039977	·000436	·261811	·000691	·301788	·000256	·281233	15° 36′	5·04826	·013340	·198097	120
125	585·66	17·6675	2544·113	22·4703	173·5996	151·3293	136·4722	14·8571	·042141	·000433	·258391	·000685	·300532	·000251	·308146	17° 3′	4·55675	·013383	·219455	125
130	590·66	19·0142	2738·044	23·4899	174·1112	150·6213	135·6848	14·9366	·044295	·000431	·255005	·000677	·299300	·000246	·324619	17° 59′	4·25757	·013427	·234875	130
135	595·66	20·6924	2979·706	24·7118	174·6112	149·8994	134·9861	15·0133	·046440	·000429	·251653	·000670	·298093	·000241	·346864	19° 8′	3·93343	·013470	·254231	135
140	600·66	22·4848	3245·197	25·9350	175·0988	149·1638	134·0766	15·0873	·048576	·000427	·248335	·000664	·296909	·000237	·370086	20° 17′	3·63885	·013514	·274812	140
145	605·66	24·3950	3512·884	27·1625	175·5770	148·4145	133·2561	15·1584	·050700	·000425	·245046	·000657	·295746	·000233	·394362	21° 31′	3·37070	·013558	·296674	145
150	610·66	26·4298	3805·884	28·3913	176·0428	147·6515	132·4747	15·2268	·052816	·000423	·241796	·000650	·294612	·000227	·419685	22° 46′	3·12627	·013602	·319870	150
155	615·66	28·5937	4117·486	29·6223	176·4970	146·8747	131·5824	15·2923	·054921	·000422	·238565	·000646	·293489	·000224	·446071	24° 2′	2·90312	·013645	·344435	155
160	620·66	30·9636	4448·499	30·8556	176·9398	146·0842	130·7294	15·3548	·057023	·000420	·235372	·000638	·292394	·000218	·473539	23° 20′	2·69910	·013689	·370496	160
165	625·66	33·2694	4790·791	32·0912	177·3712	145·2800	129·8656	15·4144	·059112	·000418	·232202	·000633	·291315	·000216	·501171	26° 37′	2·51695	·013733	·39731	165
170	630·66	35·8340	5160·100	33·3320	177·7941	144·4621	128·9915	15·4706	·061194	·000416	·229066	·000627	·290260	·000213	·522336	27° 58′	2·34597	·013776	·42626	170
175	635·66	38·6503	3565·650	34·5692	178·1966	143·6304	128·1062	15·5241	·063267	·000415	·225955	·000622	·289222	·000207	·562546	29° 22′	2 18393	·013820	·45789	175
180	640·66	41·5399	5981·748	35·8116	178·5966	142·7850	127·2107	15·5743	·065331	·000413	·222872	·000617	·288203	·000204	·594417	30° 44′	2·03950	·013864	·49032	180
185	645·66	44·4422	6399·672	37·0529	178·9787	141·9258	126·3176	15·6082	·067391	·000412	·219815	·000112	·287206	·000200	·625921	32° 1′	1·91381	·013907	·52251	185
190	650·66	47·8194	6885·992	38·2997	179·3525	141·0528	125·3869	15·6559	·069437	·000409	·216784	·000705	·286221	·000197	·661722	33° 30′	1·78393	·013951	·56056	190
195	655·66	51·1582	7366·588	39·5487	179·7151	140·1664	124·4593	15·7071	·071478	·000408	·213779	·000601	·285257	·000192	·698251	34° 51′	1·67188	·013995	·59778	195
200	660·66	54·7921	7890·056	40·8041	180·0701	139·2660	123·5216	15·7444	·073512	·000407	·210798	·000596	·284310	·000189	·733582	36° 16′	1·56636	·014039	·63822	200
205	665·66	58·5537	8431·729	42·0536	180·4555	138·3519	122·5728	15·7791	·075538	·000405	·207842	·000591	·283389	·000186	·771268	37° 39′	1·47003	·014082	·68013	205
210	670·66	62·7951	9042·500	43·3141	180·7381	137·4241	121·6141	15·8100	·077556	·000404	·204909	·000587	·282465	·000183	·813869	39° 9′	1·39437	·014126	·72765	210
215	675·66	66·6581	9598·762	44·5726	181·0552	136·4826	120·6450	15·8376	·079566	·000402	·201999	·000580	·281567	·000178	·850182	40° 22′	1·29784	·014170	·77050	215
220	680·66	71·0202	10225·485	45·8330	181·3604	135·5274	119·6657	15·8617	·081572	·000401	·199112	·000577	·280684	·000173	·891392	41° 43′	1·22104	·014213	·81897	220
225	685·66	75·6890	10899·211	47·0959	181·6553	134·5584	118·5840	15·8700	·083568	·000399	·196246	·000573	·279814	·000172	·935233	43° 5′	1·14708	·014257	·87177	225
230	690·66	80·3510	11570·539	48·3611	181·9368	133·5757	117·6766	15·8991	·085560	·000398	·193403	·000569	·278963	·000170	·977401	44° 21′	1·07338	·014300?	·93164	230
235	695·66	85·3487	12292·315	49·6286	182·2038	132·5752	116·6648	15·9995	·087545	·000397	·190575	·000565	·278120	·000168	1·02217	45° 38′	1·02165	·014344	·97881	235
240	700·66	90·5741	13042·667	50·8983	182·4673	131·5690	115·6461	15·9219	·089521	·000395	·187777	·000559	·277298	·000164	1·06824	46° 53′	·964198	·014388?	1·03713	240
245	705·66	96·0322	13828·634	52·1704	182·7155	130·5451	114·6159	15·9299	·091493	·000394	·184997	·000556	·276491	·000161	1·115264	48° 7′	·910610	·014431?	1·09817	245
250	710·66	101·7267	14648·946	53·4447	182·9522	129·5075	113·5761	15·9314	·093458	·000393	·182235	·000552	·275693	·000159	1 163647	49° 20′	·860589	·014475?	1·16199	250
255	715·66	107·6703	15540·261	54·7213	183·1774	128·4561	113·5254	15·9307	·095417	·000392	·179491	·000548	·274908	·000157	1·20479	50° 30′	·813902	·014519?	1·22865	255
260	720·66	113·8625	16396 200	56 0001	183·3911	127·3910	111·4650	15 9260	·097369	·000390	·176770	·000542	·274139	·000154	1·26383	51° 39′	·770943	·014563?	1·29849	260
265	725·66	120·3107	17234·716	57·2812	183·5934	126·3122	110·3252	15·9170	·099315	·000389	·174067	·000441	·273382	·000151	1·315692	52° 46′	·729490	·014607?	1·37086	265
270	730·66	127·0212	18091 055	58·5647	183·7845	125·2196	109·3152	15·9044	·101256	·000388	·171378	·000538	·272634	·000149	1·368728	53° 51′	·691136	·014650?	1·44689	270
275	735·66	133 9993	19295·942	59·8504	183 9638	124·1134	108·2255	15·8880	1 13192	·000387	·168710	·000533	·271900	·000146	1·429919	54° 54′	·655284	·014694?	1·52606	275
280	740·66	141·2527	20340·384	61·1383	184·1317	122·9934	107·1260	15·8674	·105120	·000386	·166059	·000530	·271179	·000144	1·478267	55° 55′	·621652	·014737?	1·60863	280
285	745 66	148 7840	21424·906	62·4286	184·2882	121·8596	106·0136	15·8430	·107043	·000385	·163425	·000526	·270468	·000141	1·534753	56° 55′	·590386	·014781?	1·69468	285
290	750.66	156·6020	22550·731	63·7213	184·4324	120·7112	104·8668	15·8144	·108962	·000384	·160807	·000523	·269769	·000139	1·592395	57° 58′	·560420	·014825?	1·78439	290
293	753·66	164·7103	23717·278	65·0160	184·5668	119·5509	103·7690	15·7819	·110874	·000382	·158207	·000520	·269081	·000137	1·651151	58° 48′	·532544	·014868?	1·87779	295
300	760 66	172·8305	24887 598	66 3130	184·6890	118·3760	102·6308	15·7452	·112781	·000381	·155623	·000516	·268404	·000135	1·708922	59° 30′	·507004	·014912?	1·972237	300
305	765·66	181 8221	26182·384	67·6123	184·7996	117·1873	101·4829	15 7044	·114683	·000380	·153054	·000513	·267739	·000133	1·772028	60° 39′	·481606	·014956?	2.07630	305
310	770.66	190·8359	27480 323	68·9140	184·8989	115·9849	100·3220	15 6629	·116580	·000371	·150501	·000510	·267081	·000131	1·834106	61° 24′	·458333	·015000?	2·18182	310
315	775.66	200·1658	28823·747	70·2179	184·9866	114·7689	99·1587	15.6700	·118467	000·377	·147963	·000507	·266430	·000130	1·897304	62° 12′	·436384	·015043?	2·29156	315


TEMPERATURE ENTROPY
DIAGRAM
CARBON DISULPHIDE
PLATE VI
TEMPERATURE IN DEG. F. H.R.
PRESSURE IN LBS. ABSOLUTE
ENTROPY
VOLUME IN CUBIC FEET
CONSTANT TEMPERATURE LINES
°F
T.M. GARDNER


TEMPERATURE - ENTROPY
DIAGRAM
FOR
CARBON - DISULPHIDE
CONSTANT AREA LINES
P V · C
PLATE III
POUNDS PER SQ INCH
DEG FAHR
LBS
CONSTANT AREA LINES
ENTROPY
CUBIC FEET
T. M. GARDNER


TEMPERATURE ENTROPY
DIAGRAM
FOR
CARBON-DISULPHIDE
CONSTANT QUALITY LINES
X·C
PLATE IV
POUNDS PER SQ. INCH.
DEGREES FAHR.
PER CENT
10 20 30 40 50 60 70 75 80 85 90 95 100
CONSTANT QUALITY LINES
SATURATION CURVE
CONSTANT QUALITY EXPANSION CURVES
90%
80%
70%
60%
50%
40%
30%
20%
10%
T.M. GARDNER
ENTROPY
CUBIC FEET


TEMPERATURE-ENTROPY
DIAGRAM
FOR
CARBON-DISULPHIDE
CONSTANT ENTROPY LINES
P·C
PLATE V
POUNDS PER SQ. INCH.
DEG. FAHR.
CONSTANT ENTROPY LINES
SUPERHEATED
C S2
SATURATION CURVE
ADIABATICS
T. M. GARDNER
ENTROPY
CUBIC FEET


TEMPERATURE ENTROPY DIAGRAM FOR CARBON DISULPHIDE
CONSTANT VOLUME LINES
PLATE II
.1 CUBIC. 2 FEET PER 3 POUND 4
.5
.6
.7
.8
.9
1
1.25
1.5
1.75
2
2.5
3
3.5
4
4.5
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
φ_l
φ_v
CONSTANT VOLUME CURVES
IN CUBIC FT. PER LB.
DEGREES FAHR.
300
290
280
270
260
250
240
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
60
50
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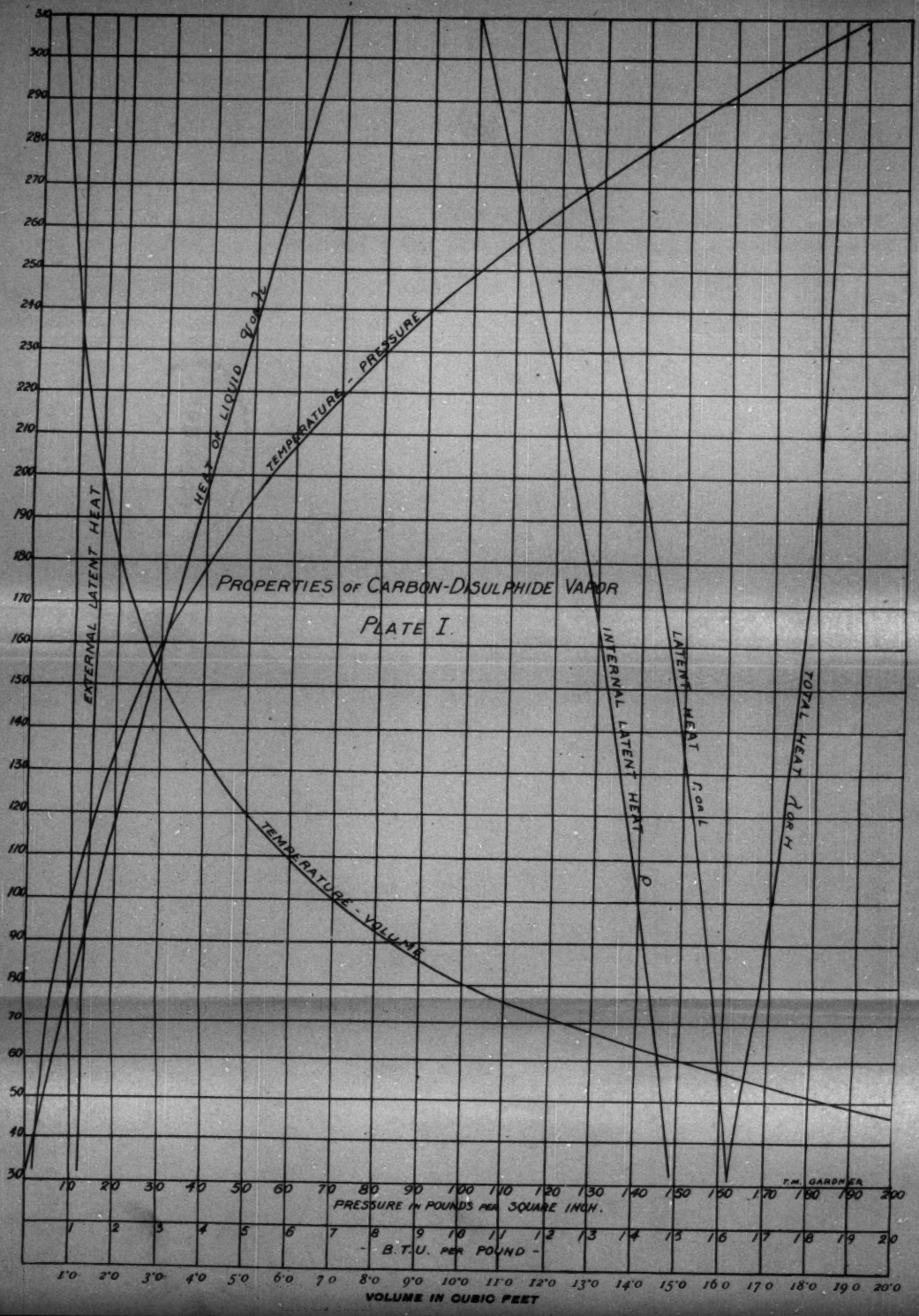
ENTROPY.


PROPERTIES OF CARBON-DISULPHIDE VAPOR
PLATE I.
EXTERNAL LATENT HEAT
HEAT OF LIQUID q or t
TEMPERATURE - PRESSURE
INTERNAL LATENT HEAT ρ
LATENT HEAT r or L
TOTAL HEAT J or H
TEMPERATURE - VOLUME
T.M. GARDNER
PRESSURE IN POUNDS PER SQUARE INCH.
- B.T.U. PER POUND -
VOLUME IN CUBIC FEET

of the liquid curve to the adiabatic and dividing by the distance ϕ_v.

In Plate V a pressure of 101 pounds absolute is taken and divided into ten equal parts, giving qualities ranging from 10 to 100 per cent. Letting the different mixtures of the fluid and its vapor expand adiabatically, we see that the amount of contained moisture increases for all qualities above 70 per cent. and decreases for all values of the dryness fraction below 60 per cent. Beginning at a temperature of 250° F. and expanding down to 50° adiabatically, if the same quality is desired at the end as at the beginning, a value of 64·5 per cent. must be taken. Starting with 100° of superheat above 250° saturation and expanding in the same manner, the final temperature will be 198° when the superheat has disappeared. It may further be said that when we compress saturated vapor of carbon disulphide it becomes an imperfect gas.

The following table will show the values of n when the fluid expanding adiabatically has the initial quality modification. These, if plotted on cross-section paper, give a logarithmic curve, which becomes plain when drawn on logarithmic cross-section sheet, when it gives a straight line. Solving, we have the equation

$$xn^{-6\cdot36} = c, \text{ or } x \div n^{6\cdot36} = c;$$

whence

$$n = \left(\frac{x}{c}\right)^{\frac{1}{6\cdot36}}$$

Plate VI.—This Boulvin θ Φ chart is for the purpose of quickly transferring the indicator diagram, representing the pressure-volume history of 1 pound of CS_2 in the engine cylinder, to the temperature-entropy quadrant and thus give us the desired heat representation.

Equations of Carbon Disulphide.

$$\log p = A - \frac{B}{\tau} - \frac{C}{\tau^2}$$

$$\tau = 1 \div \left[\sqrt{\left(\frac{A - \log p}{C} + \frac{B^2}{4C^2}\right)} - \frac{B}{2C}\right]$$

$$A = 7{\cdot}4263 \quad \log B = 3{\cdot}3274293 \quad \log C = 5{\cdot}1344146$$

$$\frac{dp}{dt} = 2{\cdot}30258\ p\left(\frac{B}{\tau^2} + \frac{2C}{\tau^3}\right) \quad \left[\frac{dp}{dt} = \tfrac{1}{144} \text{ of this value in table.}\right]$$

$$q = -\ 7{\cdot}480711 + 0{\cdot}232314\ T + 0{\cdot}00004555\ T^2$$

$$\frac{dq}{dt} = c = 0{\cdot}232314 + 0{\cdot}0000911\ T$$

$$\varphi_1 = (c)\ (2{\cdot}30258) \log 10\ \frac{\tau_1}{492{\cdot}66}$$

$$r = 164{\cdot}57 - 0{\cdot}0716\ T - 0{\cdot}0002746\ T^2$$

$$\mu = 144\left(\frac{778r}{\tau\frac{dp}{dt}}\right)$$

$$v = \mu + v_1$$

$$v_1 = {\cdot}0122903 + 0{\cdot}0000874\ T$$

Volume of 1 pound of superheated CS_2 under standard conditions of pressure and temperature = 4.7189 cubic feet, and 1 pound weight of CS_2 gas is 2·6258 times the volume of an equal amount of air.

$$pv = 20{\cdot}271\ \tau$$

k_p = ·1596 (176° F. — 374° F.). Regnault.

k_v = 1·33545 } Computed.
γ = 1·1958 }

Exponent of saturation curve, n = 1·058.

Boiling point, 115·88° F. { Thorpe. Freidburg.

Critical temperature, 504·5° F. Pressure, 978 pounds absolute.

ADIABATIC EXPANSION.

Temperature Range F°	Pressure Range Lbs.	Values of n for Qualities of									
		100 per cent.	90 per cent.	80 per cent.	70 per cent.	60 per cent.	50 per cent.	40 per cent.	30 per cent.	20 per cent.	10 per cent.
265°—190°	120·31 to 9·097	1·1155	1·1002	1·0823	1·0611	1.0355	1·0038	·96355	·9095	.83518	.7157
190°—9 °	47·82 to 9·697	1·1292	1·1154	1·1099	1·0791	1·052	1·0230	·9818	·9240	·8402	.6987
205°—120°	58·55 to 15·98	1·1236	1·1077	1·0898	1·0675	1·0367	1·0059	·9578	·8969	·8036	·65095
Averages		1·1221	1·1078	1·0904	1·0692	1·0414	1·0109	·9677	·9101	·8293	·6885

x	n
90 per cent.—50 per cent.	—6·4214
70 per cent.—50 per cent.	—6·001
100 per cent.—50 per cent.	—6·65
Average	—6·36

$$x\, n^{-6\cdot36} = c$$

$$n = \sqrt[6\cdot36]{\frac{x}{c}}$$

THE IRON PILLAR OF DELHI.

An illustration of the famous iron pillar of Delhi, India, is given in a recent number of *Cassier's Magazine*, together with the following particulars of it:

The pillar is a solid shaft of wrought iron, 16 inches in diameter, and of a length that is variously reported. The total height above ground is 22 feet, including a capital of 3½ feet. Major G. A. Cunningham, R.E., C.S.I., Director-General of the Archæological Survey of India, reported in 1863 that the total length of the pillar is upward of 48 feet, and possibly 60 feet.

The lower diameter is 16·4 inches and the upper diameter 13·5 inches. The pillar contains about 80 cubic feet of metal and weighs about 17 tons. The metal was for a long time reputed to be bronze, owing probably to a curious yellowish shade on the upper part. A sample from near the base was analyzed by Dr. Murray Thompson and found to be pure malleable iron of 7·66 specific gravity. The metal is, of course, charcoal iron, made directly from the ore in small billets; but how it was welded up no one can tell, as no record exists of any early method of dealing with great masses of wrought iron.

An inscription, roughly cut or punched upon the column, states that Raja Dhara subdued a people in the Surdhu, named Vahlikos, and obtained with his own arm an undivided sovereignty on the earth for a long period. The date of the inscription has been referred to the third or fourth century after Christ; but on this authorities are at variance, as the style of writing is supposed by some to belong to a later period.

According to tradition, the Iron Pillar was erected by Bilan Deo, the founder of the Tomara dynasty, who was assured by a learned Brahmin that, as a part of the pillar had been driven so deep into the ground that it rested on the head of Vasuki, King of Serpents, who supports the earth, it was now immovable, and that dominion would remain in his family as long as the pillar stood. The Raja, doubting the truth of the Brahmin's statement, ordered the pillar to be dug up, when the foot was found to be wet with the blood of the serpent king. The pillar was again raised, but owing to the Raja's incredulity, no means could be found to fix it firmly, and it remained loose (dhila) in the ground, and this is said to have been the origin of the name of the ancient city of Delhi.

Mining and Metallurgical Section.

Stated Meeting, held March 28, 1903.

A Study of Alloys Suitable for Bearing Purposes.

By G. H. Clamer,
Vice-President Ajax Metal Company, Member of the Institute.

DISCUSSION.

(*Concluded from page 77.*)

Dr. C. B. Dudley, Chemist to the Pennsylvania Railroad:—I have been very much interested in reading over the work done by Mr. Clamer, the results of which are given in the above paper, and I think he is to be congratulated. While it hardly seems possible to say that a finality has been reached in the study of bearing metal alloys, we are clearly of the opinion that Mr. Clamer's work in developing the plastic bronze is a decided step forward. His demonstration of the value in bearing metals of a diminution of tin and an increase of lead, and the means by which he accomplishes this successfully, must be regarded as valuable contributions to our knowledge of the subject. It was hoped that the plastic bronze might prove as great a mitigation of the hot-box trouble as the introduction of the lead lining did some years ago, and while positive tests do not seem to absolutely confirm this hope, it is not too much to say that plastic bronze is a decided improvement in this respect over any other bearing metal with which we are conversant. The value of the plastic bronze in the loss of metal by wear, and the slow wearing of the journal, seems to put it, taken all in all, ahead of any other bearing metal, and indeed ahead of the lead-lined bearing. It seems probable that further study, with the appliances now at hand, including especially the microscopic examination of metals and reliable testing machines for determining rate of wear and friction, may in the future develop something better than anything at present known, but this does not

detract from the very great value of the work done by Mr. Clamer.

MR. C. N. FORREST, Chemist to the Long Island Railroad Company:—An extensive study of the alloys of copper, tin and lead, etc., could hardly be attempted by the testing department of the ordinary railroad, and it is from such useful data as Mr. Clamer has presented that the standards of many roads have been adopted.

An alloy of practically the composition of Dr. Dudley's Ex. B. metal has been the standard bearing metal of the Long Island Railroad for a number of years, and clean, sound castings of this metal, properly finished and lined with ordinary lead and carefully inspected and tested before accepted, have made a record in the passenger service of this road which leaves little to be desired, as we now have practically no hot boxes on our cars. During the past year we used plastic bronze bearings, feeling that the composition was good, but chiefly on account of the lower cost.

I cannot, from a practical railroad service standpoint, quite agree with Mr. Clamer in his statement that the "first and foremost qualification is that the alloy have the best composition to make the bearings," and that "on this depend all the following essentials" (friction, wear on journal, etc.), although this is, of course, of very great importance. To the railroad the first desideratum is a bearing which will run cool—the life of the bearing is of secondary importance. The most perfect composition of the alloy is of little avail unless the mixture is uniform and clean, the bearing surface smooth and of the proper radius, and while these results should be obtained with alloys of theoretically the best composition, they are also obtained with alloys made from other formulas. Within reasonable limits the composition has little to do with causing or preventing hot boxes. With the same care in packing and mechanical details, the freedom from hot boxes depends almost entirely on the other points I have mentioned, and practical experiments have shown that bearings with several per cent. of zinc will run cool provided the casting is clean and sound, and the bearing surface true and smooth before it is lined.

I have no figures to show the life of the brasses on our road, but feel safe in saying that no brass remains in our passenger service long enough to wear out. Removal of wheels or axles, etc., necessitates the removal of the brasses, and it is impracticable to put the old brasses on new journals after the lead lining is gone, so that nearly all of the bronze portion of the brass goes into the scrap bin, which fact offsets to a great extent the improved wearing qualities of any special alloy.

In view of such service and practice, a slight improvement in the anti-frictional and wearing properties of a new alloy is not of sufficient advantage in itself to warrant any change of standards, and the chief attraction to the consumer remains in the lower cost of the new metal, which Mr. Clamer has not neglected, and rightly, to emphasize, and this will doubtless be at present the strongest argument in favor of plastic bronze.

The experience of the Long Island Railroad with plastic bronze bearings has not been sufficiently long to permit any statement as to the practical service of this alloy, but granting that it is possible to obtain castings of equal mechanical perfection with the new alloy, there appears to be no reason why it should not be as generally used as the older standards at the same cost, and more extensively so long as it is sold at a lower price.

Mr. Clamer's paper is a valuable contribution to our literature on alloys.

MR. CLAMER:—Mr. Forrest has probably not interpreted my meaning relative to the statement that the foremost qualification of the bearing alloy is that it have the proper composition, and on this depend all the other essentials.

It is very true that the best compositions often fail, owing to certain physical or mechanical defects, and the meaning I intended to convey is that the composition is fundamental. You cannot make a bad composition give good results, but you can make a good composition give bad results.

In discussing fracture, I confined myself to the characteristic structure of the alloys treated, and did not care to deal with the subject of segregations, etc., which are caused

by improper foundry practice. Some years ago I investigated this point very thoroughly, and I do not wish to discount in any way the value attached to the examination of fracture of bearings, as I have long recognized the value of such inspection, and, in fact, we carry this out in our daily work; breaking one brass out of each heat cast. It does happen sometimes that segregations will occur in perhaps one brass in a flask, sometimes in a whole flask, and not in others; but, generally speaking, if the last flask is good the whole heat is good, owing to this being poured from the dullest metal.

I thoroughly agree with Mr. Forrest in his statement that the first desideratum is that the bearing will run cool, but I do not think there is any doubt but that the softer compositions will often prevent hot boxes by conforming to the irregularities, etc., to which, if the harder compositions were subjected, would cause abrasion and heating. There is no better example of this than the lead-lined bearing. In other words, the bearing which will stand the greatest amount of ill use is the one which will give the least trouble in service.

The item of wear is also an important factor, irrespective of the fact that the bearings are seldom worn out in service. It has been shown that eight bearings of good composition on a car will lose by wear under normal conditions, from 1 to 3 pounds in 25,000 miles of travel, and irrespective of the fact that what is left of the bearing is remelted and made into new bearings, the metal actually lost and unrecoverable in a year's operation is a considerable amount.

The item of wear is probably of higher importance in locomotive practice, as the journals are operated directly upon the brass, and not in contact with a lead lining, as is the case of car equipment. The importance of having a composition which will run cool in locomotive service is also, for this reason, of high importance.

APPENDIX.

The Goldschmidt Theory of Harmony.

By Joseph W. Richards.

Read at the meeting of the Physical Section of the Institute, January 22, 1902.
(*Concluded from p. 235.*)

THE LAW OF INTER-RELATION IN MUSIC.

The diatonic scale from c to $\bar{c}$ may be expressed numerically in two ways, either by giving the relative times of vibration (Z) corresponding to each note, that is, the relative number of vibrations per second, or by giving the relative wave lengths (l), that is, the relative lengths of vibrating strings giving the various tones.

	c	d	e	f	g	a	b	$\bar{c}$
$Z =$	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
$l = \frac{1}{Z} =$	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

We may study each of these series to find their meaning in the harmonic numbers. We will take up first the ascending series.

Transposing the Z series so as to make $c = 0$ and $\bar{c} = \infty$, we obtain

	c	d	e	f	g	a	b	$\bar{c}$
$p = \frac{Z-1}{2-Z} =$	0	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	7	∞

Comparing this with the normal crystallographic series N_3, we note the absence of 3, and the interpolation of $\frac{1}{7}$ and 7. But $p = 3$ would correspond to $Z = \frac{7}{4}$, which is the vibration number of $b\flat$. Inserting this, we have

	c	d	e	f	g	a	$b\flat$	b	$\bar{c}$
$p =$	0	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	7	∞

The series is now symmetrical, with its middle point $g = 1$, which is the most important point of the series, next to the terminals, musically as well as crystallographically, for in music it is the "fifth," and is called the "dominant." This appellation has been carried over into crystallography.

An agreeable succession of notes is given by striking in series the following:

c		g	.	$\bar{c}$	:	$p=0$			1			∞
c	f	g	a	$\bar{c}$	:	$p=0$		$\frac{1}{2}$	1	2		∞
c	e	g	$b\flat$	$\bar{c}$	:	$p=0$	$\frac{1}{3}$		1		3	∞
c	d	g	b	$\bar{c}$	:	$p=0\frac{1}{7}$			1			7∞

The dominant, $p=1$, plays in the development of crystal forms, as in harmonic tones, a prominent and a similar part: in music, for the developmentof the structure (Fortbildung der Tonarten); in crystallography, for the building up of secondary and tertiary zones.

The numbers $\frac{1}{7}$ and 7, corresponding to the tones d and b, do not fit into or form part of the normal N_3 series. They do not appear to belong normally to that series; and in reality they are not members of the harmonic combination $c, g, \bar{c}$ (C major), but of the next related combination $g, \bar{d}, \bar{g}$ (G major). Of this latter combination, $\bar{d}$ is the "dominant" or "quint," and b is the important element, the "third." These facts are known musically; proof of them from the standpoint of the "law of inter-relation" is as follows:

Take the vibration frequencies of the series from g to $\bar{g}$, and they are:

	g	a	$b\flat$	b	$\bar{c}$	$\bar{d}$	$\bar{e}$	$\bar{f}$	$\bar{g}$
$Z=$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{7}{4}$	$\frac{15}{8}$	2	$\frac{9}{4}$	$\frac{5}{2}$	$\frac{8}{3}$	3

Transposing, with $g=0, \bar{g}=\infty$, and $p=\dfrac{Z-\frac{3}{2}}{3-Z}$, we have

	g	a	$b\flat$	b	$\bar{c}$	$\bar{d}$	$\bar{e}$	$\bar{f}$	$\bar{g}$
$p=$	0	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	$\frac{7}{2}$	∞

The normal N_3 series, developed between g and $\bar{g}$, includes, therefore,

	g	b	$\bar{c}$	$\bar{d}$	$\bar{e}$	$\left(\frac{e\sharp}{3}\right)$	$\bar{g}$
$p=$	0	$\frac{1}{3}$	$\frac{1}{2}$	1	2		∞

The tone $p=3$ corresponds to $Z=2\frac{1}{8}$, which is very nearly $e\sharp$. It is further to be observed that while b and d here become important members of the series, while they appear intruders in the series $c, g, \bar{c}$, yet, conversely, a, $b\flat$ and f,

which are important members of the series $c, g, \bar{c}$, are intruders in this series. In fact, only c, e and g are common members of both combinations, being respectively 0, $\frac{1}{3}$, 1, of the C major combination and 0, $\frac{1}{4}$, 2, of the G major series; f and a are $\frac{1}{2}$ and 2 of the C major series, and b and d $\frac{1}{3}$ and 1 of the G major series.

The combination of the harmonic numbers 0, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2 of the two series $c, g, \bar{c}$ and $g, \bar{d}, \bar{g}$, give us all the notes of our diatonic scale. This scale is therefore not a harmonically developed whole, but is heterogeneous, composed of two simple series of like form superposed upon each other. These series contain all the harmonic numbers of an N_2 series, plus $\frac{1}{3}$ from the N_3 series, and therefore stand between N_2 and N_3 in development. It is also common in crystallography to find series occurring which stand in their development between two normal series, or which contain all of the elements of one series plus one or two of a higher.

The tones of the scale lacking to make up a full N_3 series are $\frac{2}{3}$, $\frac{3}{5}$ and 3. These would correspond in vibration frequencies in the series $c, g, \bar{c}$, to $Z = \frac{7}{5}$, $\frac{8}{5}$ and $\frac{7}{4}$, that is, to the tones $f\sharp$, $a\flat$ and $b\flat$. If we add these to the series $c, \bar{c}$, we have

$N_3 : p =$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{3}{2}$	2	3	∞
corresponding to $Z =$	1	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{7}{5}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{7}{4}$	2
and the tones	c	e	f	$f\sharp$	g	$a\flat$	a	$b\flat$	$\bar{c}$

This series approximates to our chromatic scale, but is not, like it, an aggregate, but is a harmonically developed whole. It is not likely that our present style of music could use such a complicated scale. Our music has developed polyphonically, by the succession of accords from the series 0, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2, 3, ∞, arranged upon varying ground-tones. The ground-tones themselves stand in a simple relationship, being first the harmonic numbers 0, ∞, then 0, 1, ∞, then 0, $\frac{1}{2}$, 1, 2, ∞. For such simple chords based on such simple progressions of the ground-tones of the chords, the differentiation within the octave up to full N_3 is too complicated. The harmonic correllation of the progressive ground-tones gives, together with the variations of the chords themselves, as great a variety of construction as we care to make use of.

[Where part-singing has not developed, and no accords exist, but only the sequence of notes, the complication within the octave may reach N_5 and even surpass it. Such one-part music has highly developed in the Orient, as among the Arabians, Indians and Japanese. Dr. Goldschmidt has observed, by studying this one-part music in its home, in Tunis, India and Japan, that where our several-part music is introduced it destroys the delicate refinements of the one-part music. The process is going on in Japan. Japanese music appears dry to us, because there are no chords, and not pure, because it has intermediate tones which our simplified development of tonal succession does not have. On the other hand, to the musically cultivated Japanese our music appears crude in its tonal successions, since the refinements of development between the tones are lacking. Our chords give him too much music at once, and spoil for him the enjoyment of his delicate, finely modulated tonal successions. Thereby it is comprehensible that he likes his own music better. This view of the subject also explains why the Japanese melodies do not stand a many-part arrangement without losing thereby their character. Because of the chords, the tonal succession alters its nature a little, and the music is thereby no longer the same: it has become European.]

A *chord* is a group of harmonic tones sounding simultaneously. *Sequence* is the same sounding in succession. The former may be called a *standing* chord, and designated thus—$c, f, a, \bar{c}$; the latter may be called a *progression*, and designated thus—$c\text{-}f\text{-}a\text{-}\bar{c}$. Certain chords are particularly pleasing to the ear, and harmonically particularly important for the construction of musical compositions; we call them "*principal chords*."

Chords are designated by the letters of their tones or by the harmonic numbers (p) and the ground-tone to which the numbers refer. Thus we write

c	e	$g = 0$	$\frac{1}{3}$	1	(c)
g	b	$d = 0$	$\frac{1}{3}$	1	(g)

Further, the same chord may be differently interpreted, as being one combination of harmonic numbers on one ground-

tone, or a different combination on another ground-tone. Thus

c	e	$g = 0$	$\frac{1}{3}$	1	(c)
g	c	$\bar{e} = 0$	$\frac{1}{2}$	2	(g)

Taking the series of harmonic numbers

$$p = 0 \quad \frac{1}{3} \quad \frac{1}{2} \quad 1 \quad 2 \quad 3 \quad \infty$$

the principal chords are the following:

0			1			∞
0		$\frac{1}{2}$		2		∞
0	$\frac{1}{3}$		1			∞
0	$\frac{1}{3}$		1		3	∞
0	$\frac{1}{3}$			2		∞

Studying these chords, for convenience, in the octave $c, g, \bar{c}$, we may say of them:

0, 1, ∞ = c, g, c̄ is the crystallographic normal series N_1, the empty fifth. It is the foundation of harmonic sub-division; however, only a double tone. The most important combination crystallographically.

0, ½, 1, 2, ∞ = c-f-g-a-c̄ is not a standing chord, but is a melodious progression. It is important because it is the principal progression upon which the ground-tones move in that sequence of chords which form the piece of music. This series is not a good standing accord because of the nearness of g to f and a, producing discord. It would be the normal series N_2.

0, ½, 2, ∞ = c, f, a, c̄ is the normal series N_2 without the dominant $1 = g$; f, g and a are too close together to agree in a standing accord; leaving g out, the chord is melodious.

0, ⅓, 1, ∞ = c, e, g, c̄ is called the *major chord*. It is, according to the harmonic numbers, the next simple group. In this chord the tones are not near enough to cause interference; therefore, this chord is superior to 0, ½, 1, 2, ∞ as a standing chord, but not as a progression. Further, 0, ½, 2, ∞ $(c) = c, f, a, \bar{c}$ is harmonically identical with $f, a, c, \bar{f} = 0, \frac{1}{3}, 1, \infty\ (f)$; therefore, the former must also be a major chord, only lowered a fifth to another ground-tone. The major chord corresponds therefore to both 0, ½, 2, ∞ and 0, ⅓, 1, ∞. That increases its harmonic importance. The

Gau	de	a	mus	i	gi	tur	ju	ve	nes	dum	su	mus	post	ju	cun	dam	ju	ven	tu	tem	nos	ha	be	bit	hu	mus
													post	mo	les	tam	se	nec	tu	tem						
c	g	g	c	a	a	a	b	c	d	b	ce	c	b	c	d	d	e	c	d	d	c	b	af	ed	e	d	c
g	e	e	g	f	f	f	g	g	g	g	g'	g	g	a	b	b	c	g	g	g	g	g	f ·	aa	c	b	c
c	c	c	e	f	f	f	d	d	d	d	eg	e	g	g	g	g	g	e	b	b	e	e	a ·	fa	g	f	e
c	c	c	e	f	f	f	d	e	b	g	c'	c	g	g	g	g	c	c	g	g	e	e	f ·	df	g	g	c
o 1	o½1	o½1	o½1	o½	o½	o½	o½1	o½12	o½1	o½1	o½1	o½1	o½	o½2	o½1	o½1	o½1	o½1	o½1	o½1	o½1	o½2	o½	·o½2	o½1	o½13	o½1
c	c	c	c	f	f	f	g	g	g	g	c	c	g	c	g	g	c	c	g	g	c	g	f	·f	c	g	c

c f g
o ½ 1
c

c g
o 1
c

c f g
o ½ 1
c

c

major chord is the most important chord in rising harmony—perhaps even of all music.

0, $\frac{1}{3}$, 1, 3, ∞ = c, e, g, b, $\bar{c}$, the symmetrically completed major chord, is harmonic both as a chord and as a sequence. The introduction of $b = 3$ makes the chord fuller, the harmony more complete. In fact, we find this chord in four-part musical pieces at particularly emphatic or important places.

0, $\frac{1}{3}$, 2, ∞ = c, e, a, $\bar{c}$ is a minor chord, for it is harmonically identical with $e, c, a = 0, \frac{\bar{1}}{3}, \bar{1}$ (e) in descending harmony, as will be explained in considering the descending wave lengths from c to $\bar{c}$. Therefore, 0, $\frac{1}{3}$, 2 is the interpretation of a minor chord where it occurs amongst the chords of a major piece, and 0, $\frac{\bar{1}}{3}$, $\bar{1}$ is the interpretation of the same chord in minor compositions.

We have so far considered the numerical series from c to $\bar{c}$ in the sense of the *ascending* vibration numbers, and deduced from them the corresponding harmonic numbers and discussed their relationships, particularly in the building up of chords; but the reciprocals of these numbers, that is, the relative wave lengths, have an equal claim to consideration, and may equally serve as the basis of a discussion of the harmonic numbers represented. We have found that the simple combinations of harmonic numbers derived from the *ascending* numbers, that is, the *rising harmony*, give us the principal major chords and one subordinate minor chord. We will find that an exactly similar treatment applied to the *descending* series of *decreasing* wave lengths, from c to $\bar{c}$, will give for the simple combination of the harmonic numbers derived therefrom the principal minor chords and one subordinate major chord.

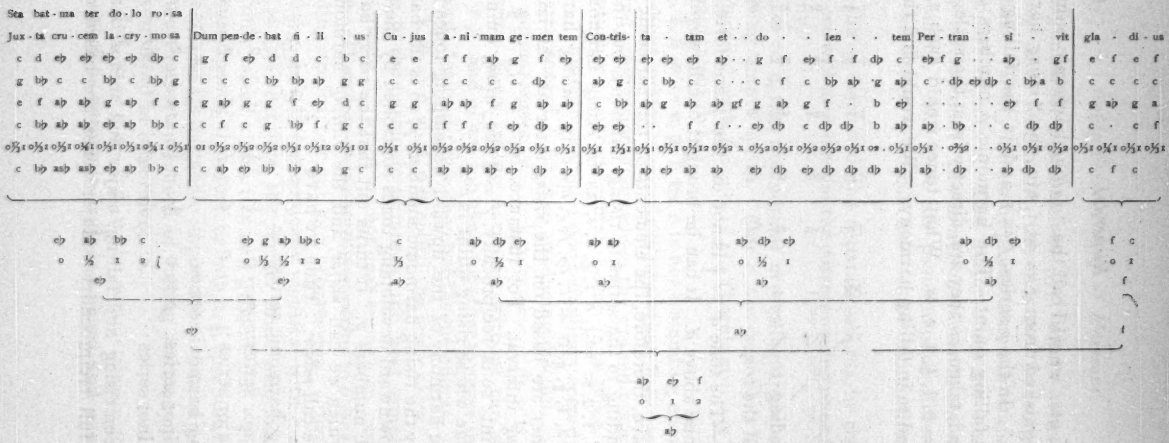
The wave lengths are the reciprocals of the vibration numbers, and are:

	c	d	e	f	g	a	b	$\bar{c}$
$Z = \frac{1}{Z} =$	1	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

and the corresponding p series (designated as $\bar{p}$ to distinguish from the reciprocal tones in rising harmony):

	c	d	e	f	g	a	b	$\bar{c}$
$\bar{p} =$	$\bar{\infty}$	$\frac{\bar{1}}{2}$	$\frac{\bar{3}}{2}$	$\bar{1}$	$\frac{\bar{1}}{2}$	$\frac{\bar{1}}{4}$	$\frac{\bar{1}}{14}$	0

Sta - bat ma-ter do - lo - ro - sa
dum pen-de-bat fi - li - us
jux - ta cru-cem la-cry- mo - sa
Cu-jus a - ni-mam ge-
et do - len - tem per - trans i - vit gla - di - us.
et do-len tem per - trans - vit gla-di - us.
et do-len- tem per-trans-i - vit gla - di - us.
et do - len - tem per - trans - i - vit gla - di - us.
men - tem con-tris-ta - tam et do len - tem per - trans-i vit gla - di - us.
con-tris-ta - tam et do-len tem per - trans - i - vit gla-di - us.
men - tem con - tris-ta-tam et do-len- tem per-trans-i - vit gla - di - us.


Sea bat - ma ter do - lo ro - sa
Jux - ta cru - cem la - cry - mo sa
Dum pen-de - bat fi - li - us
Cu - jus
a - ni - mam ge - men tem
Con-tris- ta - tam et - do - len - tem
Per - tran - si - vit
gla - di - us

$\overline{p} = \overline{\infty}, \overline{1}$, etc., we will call the "*falling* harmonic numbers," as distinguished from $p = \infty, 1$, etc., the "*rising* harmonic numbers." In the $\overline{p}$ series, $f = \overline{1}$ and is the falling dominant, the falling quint. The harmonic $\overline{p}$ numbers do not correspond, taken, as they are, directly from the scale, to the p series $= 0, \frac{1}{3}, \frac{1}{2}, 1, 2, \infty$. What tones would such a series correspond to in falling harmony? We would find them as follows:

Putting	$\overline{p} = \overline{\infty}$	$\overline{3}$	$\overline{2}$	$\overline{1}$	$\overline{\frac{1}{2}}$	$\overline{\frac{1}{3}}$	0
Then $Z = \frac{1}{\overline{Z}} = \overline{Z} =$	1	$\frac{7}{8}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{6}{8}$	$\frac{1}{2}$
corresponding to $Z =$	1	$\frac{8}{7}$	$\frac{6}{5}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{8}{5}$	2
and the tones	c	$(d^{\natural})$	$e\flat$	f	g	$a\flat$	$\overline{c}$

[N. B.—The tone $d^{\natural} = \frac{8}{7}$ is not on the piano-forte, but is $\frac{1}{56}$ of a tone above d. It can be brought out, however, on a violin.]

We have, therefore, the elements of the harmonic series corresponding to the series in rising harmony, but in place of $0, \frac{1}{3}, \frac{1}{2}, 1, 2, \infty = c, e, f, g, a, \overline{c}$, as in *rising* harmony, we have $\overline{\infty}, \overline{2}, \overline{1}, \overline{\frac{1}{2}}, \overline{\frac{1}{3}}, 0 = c, e\flat, f, g, a\flat, \overline{c}$ in *falling* harmony. The former are based on the *rising* interval; the latter on the *falling* interval. For instance, for $c = 1, g = \frac{3}{2}$ is the *rising* quint; so is, reciprocally, $f = \frac{3}{2}$ the *falling* quint, that is, the tone whose rising quint is $c = 1$.

As the harmony of the upward developing tones is controlled by the rising harmonic numbers p, so is the harmony of the downward developing tones controlled by the falling harmonic numbers $\overline{p}$. It turns out that the *rising* harmony is that which we call *major*, and the *falling* harmony that which we call *minor*. Thus we have

Rising harmonic series = Major series:	$p = 0$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	∞
e.g., *C* major series:	c	e	f	g	a	$\overline{c}$
Falling harmonic series = Minor series:	$\overline{p} = 0$	$\overline{\frac{1}{3}}$	$\overline{\frac{1}{2}}$	$\overline{1}$	$\overline{2}$	$\overline{\infty}$
e.g., *C* minor series:	$\overline{c}$	$a\flat$	g	f	$e\flat$	c

Between rising and falling, major and minor harmony, there is full reciprocality. Major tones are based on the

increasing number of vibrations (Z); minor tones on the *decreasing* wave lengths (l); the ones are the reciprocals of the others. Every harmonic phenomenon in major has its counterpart in minor. The major character lies in the *upward* development; the minor character in the *downward* development. May not the psychological differences of these two styles of music have some foundation in this fact?

We have discussed the separate chords in rising harmony; we will now discuss the corresponding ones in falling harmony.

0, $\bar{1}$, $\overline{\infty}$ = $\bar{c}$, f, c, the normal crystallographic series N_1, the falling empty quint. But $c, f = 0, \bar{1}\ (\bar{c})$ is harmonically identical with $f, \bar{c} = 0, 1\ (f)$. Therefore, 0, 1 is harmonically equivalent to 0, $\bar{1}$.

0, $\bar{\frac{1}{2}}$, $\bar{1}$, $\bar{2}$ $\overline{\infty}$ = c, g, f, e♭, c is not a standing chord, but is a melodious progression. It is important because it is the principal progression upon which the ground-tones move in that sequence of chords which occur in a piece of minor music. This series is not a good standing accord, because of the nearness of g, f and $e♭$ producing discord. It would be the normal series N_2. $0, \bar{\frac{1}{2}}, \bar{2}, \overline{\infty} = \bar{c}, g, e♭, c$ is the normal series N_2 in falling harmony without the dominant $f = 1$. f g, and $e♭$ are too close together to agree in a standing chord; leaving f out, the chord is melodious.

0, $\bar{\frac{1}{3}}$, $\bar{1}$, $\overline{\infty}$ = $\bar{c}$, a♭, f, c is called the *minor chord.* It is, according to the harmonic numbers, the next simple group. In this group the tones are not near enough to cause interference. Therefore, this chord is superior to $0, \bar{\frac{1}{2}}, \bar{1}, \bar{2}, \overline{\infty}$ as a standing accord, but not as a progression. Further, $0, \bar{\frac{1}{2}}, \bar{2}, \overline{\infty}\ (\bar{c})\ \bar{c}, \bar{g}, e♭, c$ is harmonically identical with $\bar{g}, e♭, c, g = 0, \bar{\frac{1}{3}}, 1, \overline{\infty}\ (\bar{g})$; therefore, the former must also be a minor chord, only raised a fifth to a new ground-tone. The minor chord corresponds, therefore, to both $0, \bar{\frac{1}{2}}, \bar{2}, \overline{\infty}$ and $0, \bar{\frac{1}{3}}, 1, \infty$. That increases its harmonic importance. The minor chord is the most important chord of falling harmony; it rivals the major chord in importance.

0, $\bar{\frac{1}{3}}$, $\bar{1}$, 3, $\overline{\infty}$ = $\bar{c}$, a♭, f, d^1, c, the symmetrically completed minor chord, is harmonic both as a chord and a sequence.

The introduction of $d^{\text{I}} = \overline{3}$ makes the chord fuller, the harmony richer. d^{I} is, however, only $\frac{1}{6\,6}$ tone above d, and cannot be produced on a piano.

0, $\frac{1}{3}$, 2, $\infty = \overline{\infty}$, a♭, e♭, c is a major chord, for it is harmonically identical with $a\flat$, c, $e\flat = 0, \frac{1}{3}, 1$ ($a\flat$) in ascending harmony. This major chord is to be interpreted as $0, \overline{\frac{1}{3}}, \overline{2}$ when it occurs interspersed between the chords of a composition in falling harmony (minor music), and as $0, \frac{1}{3}, 1$, where it occurs in pieces of rising harmony (major compositions).

Progression upon varying ground-tones is the key to the development of a musical composition. This is to say, that the chords $0\ 1$, $0\ \overline{1}$, $0\ \frac{1}{2}\ 2$, $0\ \overline{\frac{1}{2}}\ \overline{2}$, $0\ \frac{1}{3}\ 1$, $0\ \overline{\frac{1}{3}}\ \overline{1}$, $0\ \frac{1}{3}\ 1\ 3$, $0\ \frac{1}{3}\ 2$, $0\ \overline{\frac{1}{3}}\ \overline{2}$, which are the principal ones of the normal series N_3, may be based on, or have for their ground tones c, e, f, g, a or $\overline{c}$ in major music, or $\overline{c}$, $a\flat$, g, f, $e\flat$ or c in minor music, either one of these notes being the 0, or ground-tone, of the chord, and between it and its octave the harmonic development to ∞ or $\overline{\infty}$ may subsist and the notes corresponding to the intermediate members of the chord be determined.

Progression of the ground-tones, however, does not take place haphazard, but is determined by the same law of development which determines the harmonies of a chord. Given a chord in the series

$p = 0$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	∞
or $p = 0$	$\overline{\frac{1}{3}}$	$\overline{\frac{1}{2}}$	$\overline{1}$	$\overline{2}$	$\overline{\infty}$

then the progression of the ground-tones passes preferably to ∞ or $\overline{\infty}$, to 1 or $\overline{1}$, to $\frac{1}{2}$ and 2 or $\overline{\frac{1}{2}}$ and $\overline{2}$, or to $\frac{1}{3}$ or $\overline{\frac{1}{3}}$, which order is the order of relative importance of these derived tones. We have thus, essentially, three principal varieties of progression of the ground-tones, viz., to the octaves, to the dominants, to the intermediate tones.

Progression upon the octaves simply repeats the chords higher or lower in the scale.

Progression upon the dominant fills in our diatonic scale with notes which are lacking in the primary N_3 scale based on c. It is most important for the building up of our tonic system.

Progression upon the intermediate tones, chiefly ascending on $p = \frac{1}{3}, 2$, or falling on $p = \overline{\frac{1}{3}}, \overline{2}$, subordinately upon the

higher numbers $p = \pm \frac{1}{8}, 3$, is the field of detail concerning the composition of musical pieces the choice of which—their alternations, symmetrical arrangement, or even peculiarities of arrangement—constitutes the characteristics of the composition and often marks the individuality of the composer.

KEY TO THE CHORDS.

MAJOR CHORDS.

Harmonic numbers rising $p =$	0	$(\frac{1}{7})$	$(\frac{1}{4})$	$\frac{1}{3}$	$\frac{1}{2}$	1	$(\frac{3}{2})$	2	3	∞
	c	*d*	**e♭**	**e**	**f**	**g**	**a♭**	**a**	**b♭**	**c**
	c♯	*d♯*	*e*	**e♯**	**f♯**	**g♯**	*a*	**a♯**	*b*	**c♯**
	d♭	*e*	*f♭*	*f*	*g♭*	*a♭*	*a*	*b♭*	*c♭*	*d♭*
	d	**e♯**	**f**	**f♯**	**g**	**a**	**b♭**	**b**	*c*	**d**
	d♯	*f*	*f♯*	*g*	*g♯*	*a♯*	*b*	*c*	*c♯*	*d♯*
	e♭	*f*	*g♭*	*g*	*a♭*	*b♭*	*b*	*c*	*d♭*	*e♭*
	e	*f♯*	**g**	**g♯**	**a**	**b**	**c**	**c♯**	*d*	**e**
	f	*g*	**a♭**	**a**	**b♭**	**c**	**d♭**	**d**	*d♯*	**f**
	f♯	*g♯*	*a*	*a♯*	*b*	*c♯*	*d*	*d♯*	*e*	*f♯*
	g♭	*a♭*	*a*	*b♭*	*b*	*d♭*	*d*	*e♭*	*e*	*g♭*
	g	*a*	**b♭**	**b**	**c**	**d**	**e♭**	**e**	*f*	**g**
	g♯	*a♯*	*b*	*c*	*c♯*	*d♯*	*e*	*e♯*	*f♯*	*g♯*
	a♭	*b♭*	*c♭*	*c*	*d♭*	*e♭*	*e*	*f*	*g♭*	*a♭*
	a	*b*	**c**	**c♯**	**d**	**e**	**f**	**f♯**	*g*	**a**
	a♯	*c*	*c♯*	*d*	*d♯*	*e♯*	*f♯*	*g*	*g♯*	*a♯*
	b♭	*c*	*d♭*	*d*	*e♭*	*f*	*g♭*	*g*	*a♭*	*b*
	b	**c♯**	**d**	**d♯**	**e**	**f♯**	**g**	**g♯**	*a*	**b**
	c	**d**	*e♭*	**e**	**f**	**g**	**a♭**	**a**	*b*	**c**

Harmonic numbers falling $\overline{p} =$	$\overline{\infty}$	$\overline{3}$	$\overline{2}$	$\overline{(\frac{3}{2})}$	$\overline{1}$	$\overline{\frac{1}{2}}$	$\overline{\frac{1}{3}}$	$\overline{\frac{1}{4}}$	$\overline{\frac{1}{7}}$	$\overline{0}$

MINOR CHORDS.

In order to facilitate the dissection and analysis of musical compositions according to the principles so far developed, it is convenient to have a tabular arrangement of the different chords, giving the designation of each tone when considered in any chord based on any ground tone, either in rising or in falling harmony. The following is an anglicized copy of the table constructed by Dr. Goldschmidt.

IDENTIFICATION OF A CHORD BY MEANS OF THE KEY.—The key gives us in the right-hand direction the major harmonies, characterized by the harmonic numbers

$$p = 0 \quad (\tfrac{1}{4}) \quad \tfrac{1}{3} \quad \tfrac{1}{2} \quad 1 \quad 2 \quad 3 \quad \infty$$

taken in the left-hand direction the minor harmonies, characterized by

$$\bar{p} = 0 \quad \overline{(\tfrac{1}{4})} \quad \bar{\tfrac{1}{3}} \quad \bar{\tfrac{1}{2}} \quad \bar{1} \quad \bar{2} \quad \bar{3} \quad \overline{\infty}$$

We may, therefore, read off for any chord its harmonic numbers according to the ground-tone chosen. Every chord, however, will be found to have *four interpretations*—two in major harmony and two in minor harmony. The major chord *c, e, g*, for instance, may be regarded either as

$c\ e\ g = 0\ \tfrac{1}{3}\ 1$ based on ground-tone $c = 0\ \tfrac{1}{3}\ 1\ (c)$ } rising or
$g\ c\ e = 0\ \tfrac{1}{2}\ 2$ " " " " $g = 0\ \tfrac{1}{2}\ 2\ (g)$ } major.

$e\ c\ g = 0\ \bar{\tfrac{1}{3}}\ \bar{2}$ " " " " $e = 0\ \bar{\tfrac{1}{3}}\ \bar{2}\ (e)$ } falling or
$g\ e\ c = 0\ \bar{\tfrac{1}{4}}\ \bar{1}$ " " " " $g = 0\ \bar{\tfrac{1}{4}}\ \bar{1}\ (g)$ } minor.

As another instance, the minor chord *a, f, d* may be interpreted as either

$a\ f\ d = 0\ \bar{\tfrac{1}{3}}\ \bar{1}$ based on ground-tone $a = 0\ \bar{\tfrac{1}{3}}\ \bar{1}\ (a)$ } falling or
$d\ a\ f = 0\ \bar{\tfrac{1}{2}}\ \bar{2}$ " " " " $d = 0\ \bar{\tfrac{1}{2}}\ \bar{2}\ (d)$ } minor.

$f\ a\ d = 0\ \tfrac{1}{3}\ 2$ " " " " $f = 0\ \tfrac{1}{3}\ 2\ (f)$ } rising or
$d\ f\ a = 0\ \tfrac{1}{4}\ 1$ " " " " $d = 0\ \tfrac{1}{4}\ 1\ (d)$ } major.

The question thus arises, what governs the interpretation in any given case; that is, which of the four possible interpretations shall be given any given combination or chord? The conclusions on this point arrived at by Goldschmidt are as follows:

(1) The simplest interpretation is primarily the most probable one. As a rule, *c*, *e*, *g* is to be regarded as 0, $\frac{1}{3}$, 1 (*c*); *a*, *f*, *d* as 0, $\frac{1}{3}$, $\bar{1}$ (*a*).

(2) A homogeneous piece of music is to be regarded as either in rising harmony (major) or in falling harmony (minor). If the major chords predominate, the rising interpretation is to be preferred; if the minor chords predominate, the falling interpretation is to be preferred. If major and minor chords are equally present, then either a minor or a major interpretation may be given to the piece.

(3) Throughout a composition, or at least a completed part of the same, the one interpretation chosen must be adhered to; no change can be made inside such a part.

(4) If, with a major interpretation, minor chords occur interspersed between the major chords 0, $\frac{1}{3}$, 1, they are in general to be regarded as the major chords 0, $\frac{1}{3}$, 2, or, exceptionally, as 0, $\frac{1}{2}$, 1. If, with a minor interpretation, major chords occur interspersed between the minor chords 0, $\frac{1}{3}$, $\bar{1}$, they are in general to be regarded as the minor chords 0, $\frac{1}{3}$, $\bar{2}$, or, exceptionally, as 0, $\frac{1}{2}$, $\bar{1}$.

The foundation of these rules lies in the fact that the ground tone assigned to the chord varies with the interpretation given to it. But the ground-tones stand in relation to each other; their progression is the progressive harmony of the composition. If the above rules are adhered to, it will be found that the ground-tones are related to each other according to simple progressions of harmonic numbers, such as on 0, $\frac{1}{2}$, 1, 2, or 0, $\bar{\frac{1}{2}}$, $\bar{1}$, $\bar{2}$, and, rarely, on 0, $\frac{1}{3}$, $\frac{1}{2}$, 1, 2. If the rules given are not adhered to, this regularity of progression of the ground-tones will be interfered with or obliterated. This fact is the justification for the rules of interpretation given.

As examples of the application of Goldschmidt's method, we reproduce from his monograph the analysis of a very simple well-known piece, the "Gaudeamus Igitur," and that of Palestrina's "Stabat Mater," whose intricacies Helmholtz had declared to be incomprehensible from the standpoint of musical science.

Notes and Comments.

CROOKES ON ATOMIC FOG.

Prof. Sir William Crookes, before the International Chemical Congress in Berlin, recently, dealt with the possibility of reducing all the elements of matter to one, and ultimately finding this resolvable into a single form of energy. The subject was "Modern Views on Matter—the Realization of a Dream." Sir William cited the utterances of Sir Humphrey Davy and Faraday as anticipating the possibility of reducing the elements to simpler bases, spoke of the significance of the Röntgen rays and Becquerel rays, and the experiments of Curie and others, and said:

"All these observations find internal connection in the discovery of radium, which is probably the basis of the coarser chemical element. Probably masses of molecules dissolve themselves into the ether waves of the universe, or into electrical energy. Thus we stand on the border line where matter and force pass into each other. In this borderland lie the greatest scientific problems of the future. Here lie the final realities, wide-reaching and marvelous.

"The nineteenth century saw the birth of new views regarding the nature of atoms, electricity and ether. While our views about the composition of the matter are generally satisfactory to-day, will that be the case at the end of the twentieth century? Do we not again see that our investigations have only a temporary value? Will we be content to see matter dissolving into a multitude of revolving electrodes? Such a mysterious dissolution of atoms appears to be universal. It occurs when a piece of glass is rubbed with silk. It is present in sunshine, in a raindrop, in lightning, in a flame, in a waterfall, and in the roaring sea. Although the whole range of human experience is too short to form a parallax whereby we can foretell the disappearance of matter, nevertheless it is possible that formless nebulæ will again prevail when the hour glass of eternity has run out."—*Electrical World.*

RECOVERY OF TIN FROM TIN-SCRAP.

Some interesting details relating to the recovery of tin from tin-scrap have recently been published in the *Zeitschrift f. Elektrochemie.* In Germany, several works have been built and operated for carrying out this procedure the largest of these being that of Goldschmidt, at Essen, where 50 to 60 tons of tin-scrap are reported to be treated per day. The difficulty of obtaining an adequate supply of raw material has hindered the development of other works, and the anonymous writer of the article we are discussing hints that the supply of tin-scrap is monopolized by one or two of the larger works. The processes used for recovering the tin are based upon the use of the scrap as anode material, in a bath containing sodium chloride and hydrate, or in one containing hydrochloric acid. The advantage of the former is that less iron goes into solution, but against this there is the lower efficiency of the process and the more spongy nature of the deposit obtained at the kathode. The

failure to produce directly metallic tin is one of the chief difficulties in the operation of both processes, for considerable losses occur in smelting and refining the spongy deposit obtained at the kathode. Under certain conditions metallic tin can be obtained in the electrolytic bath, and Pfanhauser, in the issue of the *Zeitschrift f. Elektrochemie* for January 16th, has stated his opinion that the avoidance of formation of sponge is simply a question of maintaining the *concentration of the tin salt solution* in the neighborhood of the kathode. This condition would appear to be difficult to attain in the works treating tin-scrap on an industrial scale, and the problem of producing metallic tin at the kathode is complicated further, by the slow but gradual increase of impurities in the electrolyte. A new works for the treatment of tin-scrap has recently been built at Pfaffstätten, near Vienna, and an electrolytic process for recovering tin from slags is also reported to be in operation at Tostedt, in Germany.—*Scientific American Supplement.*

BAUXITE AND ITS USES.

The *Elektrochemische Zeitschrift* contains a useful reference article by Dr. H. Lienau on bauxite. This mineral is the chief source of the commercial aluminum produced by the electrolytic processes, and although many attempts have been made, and are still being made, to replace it by some cheaper raw material, these attempts hitherto have beed unsuccesful. Natural deposits of bauxite occur in France, Germany, Ireland, and the United States, those of the department Var in Southern France being at present the most extensively worked. In 1896 the district produced 29,620 tons, and in 1901, 65,000 tons, of which total 55,000 tons were exported to other countries. After a reference to the geological formation in which bauxite occurs and to the varying composition of the deposits, the author describes the various processes which have been worked at one time or another for extracting aluminum or its compounds from this source. The first patented process dates from 1858, and had for its object the extraction of aluminum hydrate from red bauxite. The demand by paper and color manufacturers for a cheaper source of aluminum sulphate than the alums turned attention to the direct production of aluminium sulphate from bauxite, but the efforts to produce this salt, free from iron, from red bauxite, have not been completely successful. The utilization of bauxite for the electrometallurgical production of aluminum is a comparatively recent development, but very large quantities of the mineral are now being used in aluminum reduction works. The author surmises from this fact that bauxite is being employed directly in the electrolytic baths, and that the troublesome and costly process by which alumina was first extracted from the raw bauxite is now being dispensed with.

[This surmise is most probably erroneous. Bauxite is now being refined in the electric furnace, on the large scale, for the production of alumina in the manufacture of the metal, and the tedious and costly chemical refining process is thereby avoided. A previous refining treatment of the bauxite is in all cases necessary, to remove the impurities (iron and silica), which otherwise would so contaminate the resulting metal as to render it practically worthless.—ED.]

The Generation of Electric Pressures, Types of Dynamos, The Electric Motor, Efficiencies and Dynamo Calculations, Systems of Electrical Units, Useful Tables and Constants.

The author has endeavored to meet the wants of students of electrical engineering by the introduction of numerous—nearly 700—exercises. W.

Chemistry, Inorganic and Organic, with Experiments. By Chas. Loudon Bloxam. Ninth edition, rewritten and revised. By John Millar Thompson, LL.D., F.R.S., Professor of Chemistry, King's College, London, and Arthur G. Bloxam, F.I.C., Consulting Chemist, etc. Large 8vo, pp. xi + 872. Philadelphia: P. Blakiston's Son & Co. 1903. Price, $6.00, net.

The abovenamed work is a new edition of a longtime standard text-book revised and brought up to date. A number of changes have been introduced in the treatment of the subject to conform to modern views, but the general characteristics that have made the work so popular have been retained. This is especially noticeable in the constant reference to experiments to illustrate the subject. W.

Engineering Preliminaries for an Interurban Electric Railway. By Ernst Gonzenbach. (8vo, pp. 71). New York: McGraw Publishing Company. 1903.

This work gives a careful exposition of the technical details needful for the engineer or exploiter of such enterprises, for the construction of an interurban electric railway. The problem has been apparently thoroughly studied and worked out on very conservative lines, and the work should prove extremely useful for its intended purpose. W.

Franklin Institute.

[*Proceedings of the stated meeting held Wednesday, September 16, 1903.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, September 16, 1903.

Vice-President WASHINGTON JONES in the chair.

Present, 19 members and visitors.

Additions to membership since last report, 32.

Mr. Frank C. Watson, Assistant Superintendent of Bridges, Bureau of Highways, Philadelphia, exhibited and described a series of photographic views of a large number of the bridges maintained by the municipality. The structures shown and described were chiefly the minor bridges.

The speaker was given a vote of thanks at the close of his remarks.

Adjourned,

WM. H. WAHL,
Secretary